Environmental Policy and Directed Technical Change in a Global Economy: The Dynamic Impact of Unilateral Environmental Policies.

David Hémous*

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Abstract

In an open economy, can unilateral environmental policies undertaken by a group of committed countries ensure sustainable growth? This paper addresses this question in a dynamic model with directed technical change. There are two countries and two tradeable goods. One of the two goods (the polluting good) is produced with a clean input and a dirty input, which causes a global externality. Innovation can be targeted at both sectors and, within the polluting sector, at clean or dirty technologies. For most of the analysis, innovation is local. I show that carbon taxes in a single country are generally unable to ensure sustainable growth, that is, to prevent environmental quality from falling below some critical threshold. A temporary combination of clean research subsidies and a tariff in a single country can ensure sustainable growth for sufficiently large initial quality of the environment. Both trade and directed technical change accelerate environmental degradation under laissez-faire or with a carbon tax in a single country, but both help reducing environmental degradation when the appropriate unilateral policy is undertaken. The first best policy and the second best policy when one country is constrained to be in laissez-faire are characterized both analytically and numerically in calibrated simulations. Finally, I show how the main results of the paper are robust to the inclusion of cross-country knowledge spillovers.

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1 Introduction

Countries not subject to any binding constraints under the Kyoto protocol account for an increasing fraction of CO_2 emissions; their share in world emissions has risen from a third in 1990 to more than a half in 2008. In the meantime, climate negotiations have stalled, and no global agreement seems to be in sight. In response, several countries either have undertaken unilateral actions or are considering doing so, and more and more, these policies harbor some protectionist aspects. For instance, the American Clean Energy and Security Act, which was supposed to set up a cap-and-trade system in the US, planned to implement trade barriers with countries that did not have a similar system in the absence of an international agreement by 2018.[1] This raises two questions: First, can unilateral policies be enough to ensure sustainable growth? Second, how necessary is protectionism to achieve this goal?

Fundamentally, these questions are about the long-run behavior of the economy. Over such a time period, comparative advantages are going to evolve with innovation, while innovation itself will respond to environmental policies. Therefore, a dynamic framework is necessary, this paper builds such a dynamic model by integrating directed technical change in a trade model with a global pollution externality. It first shows that unilateral environmental policies that have no protectionist component (like a carbon tax) typically fail at ensuring sustainable growth, in fact, because of the innovation response, these policies are likely to accelerate environmental degradation. Second, the paper shows that intervening countries can achieve sustainable growth without cooperation from the rest of the world by implementing a temporary industrial policy which combines clean research subsidies and a tariff. Such a policy develops clean technologies in the polluting sector in the intervening countries, which leads to a reduction in emissions in the long-run not only in the intervening countries but also in the non-intervening ones.

More formally, I consider a dynamic version of a two country (North and South), two sector and two factor Heckscher-Ohlin model - with another factor, scientists, used for innovation. The North represents countries willing to implement an environmental policy (the intervening countries), while the South represents countries that do not undertake any policy. One sector never pollutes, while the other sector can be more or less polluting depending on the balance between dirty and clean technologies in the country. In practice, the polluting sector includes the manufacturing of chemicals and chemical products, of non-metallic mineral products and of basic metals; clean technologies may correspond to renewable energy or bioplastics. Innovation can be directed at the non-polluting or the polluting sector. Within the polluting sector, it can

[1] In this case, the trade barrier was an international reserve allowance. The bill passed the House in 2009 but was rejected by the Senate. Trade barriers have also been discussed for the European Union Emissions Trading System (EU ETS), but have not been imposed yet. However, the EU ETS has been extended to air transport in January 2012, and concerns all airlines, making it the first attempt at taxing foreign firms for pollution.

be targeted at the clean or the dirty technologies. The allocation of innovation between the two sectors depends on the relative size of both sectors in the country as measured by their revenue share (Acemoglu (1998)). Since the exporting sector has a relatively larger market size than the importing sector, innovation is tilted towards the exporting sector in comparison to the other country, which creates a force towards the amplification of comparative advantage over time. Within the polluting sector, the allocation of innovation between clean and technologies is tilted towards the most advanced of the two: there is path dependence in innovation. For the main part of the analysis, innovation is assumed to be completely local.

In laissez-faire, if clean technologies are initially less advanced than dirty ones in both countries, innovation is continually directed mostly towards the dirty technologies. Emissions keep increasing and the economy eventually reaches an "environmental disaster" as the quality of the environment falls below a critical threshold. In other words, economic growth is not sustainable. If the South initially has a comparative advantage in the polluting sector, it would tend to specialize in that sector in laissez-faire. Carbon taxes and taxes on dirty research in the North lead to reallocation of some of the production of the polluting good from the North to the South (the "pollution haven effect."), therefore they can only reinforce this specialization. Emissions still grow unboundedly in the South which leads to an environmental disaster. As the reallocation of production goes hand in hand with a reallocation of innovation towards the polluting sector in the South, and as innovation in the polluting sector in the South will be mostly directed at dirty technologies, such policies are likely to accelerate environmental degradation. In contrast, temporary clean research subsidies in the North can redirect innovation from both dirty technologies and the non-polluting sector towards clean technologies in the North. A temporary policy that combines clean research subsidies with a tariff allows the North to develop a comparative advantage in the polluting sector at the same time as this sector is becoming cleaner. Once clean technologies in the North are sufficiently advanced, and the initial comparative advantage is reversed, market forces that were previously driving the economy into a disaster now work towards averting it: emissions decrease both in the North (as innovation keeps being directed to clean technologies) and in the South (as the South specializes over time in the non-polluting sector). If the initial environmental quality is sufficiently large, an environmental disaster will be averted.[2] Directed technical change is essential for this result: with exogenous technical change, policies in the North only may fail at preventing a disaster - no matter how large the initial quality of the environment is.

I consider two objective for a social planner, one where the social planner cares only about

[2] One may think that such a reversal of comparative advantage could be ensured with clean research subsidies only, but this is not always true. Under free trade, the South may fully specialize, in which case all of its innovation will be directed towards the polluting sector, such that the North will not be able to acquire a comparative advantage in the polluting sector. A tariff can prevent such an outcome.

world consumption and environmental quality and one where he maximizes a weighted sum of the utilities of infinitely-lived representative agents in both countries (and therefore cares about the distribution of consumption across the two countries). In both cases, I characterize the first best policy and the second best policy under the constraint that no intervention can occur in the South. This second best policy can be decentralized through a carbon tax and research subsidies in the North, along with a trade tax on the polluting good.[3] Absent redistribution concerns, the trade tax typically takes the form of a tariff and then of an export subsidy; its expression reflects two objectives for the social planner: reducing emissions in the South and redirecting Southern innovation towards the non-polluting sector. Yet, when the social planner cares about the distribution of income, the trade tax also reflects terms of trade considerations.

I carry out a simple calibration exercise which illustrates the main results of the paper. It shows that for reasonable parameter values, the welfare costs of not being able to implement any policy in the South are very large. It highlights the double-edged nature of both trade and directed technical change: both accelerate environmental degradation under laissez-faire, but help reduce environmental degradation when one country intervenes. Finally, I relax the assumption that knowledge is purely local, and I assume that the more backward country can partially catch up every period. The main results still apply in this context: carbon taxes and taxes on dirty research may still fail at preventing an environmental disaster, while a combination of clean research subsidies and a carbon tariff will be able to prevent an environmental disaster for sufficiently large initial environmental quality. In this case, however, the diffusion of knowledge can ensure that a switch towards clean innovation occurs in the South, so that an environmental disaster can be prevented with the South still specializing in the polluting good in the long-run.

This paper can be interpreted as providing a "clean infant industry argument." The classic infant industry argument is that trade can be detrimental to growth by leading countries in the South to specialize in sectors with poor development prospects (Krugman (1981), Young (1991), Matsuyama (1992) and Galor and Mountford (2008)). Here as well, a country risks specializing in the "wrong" sector, not because that sector offers poor growth prospects, but because this country cannot prevent the environmental externality associated with the production in this sector. Related to this literature, Krugman (1987) studies the dynamic evolution of comparative advantage in a two country world with learning by doing. As in the present analysis, comparative advantage amplify over time in laissez-faire and temporary trade policy can have a permanent impact and change the pattern of comparative advantages. In the context of endogenous growth, the dynamics of comparative advantages has been studied by Grossman and Helpman (1991) in chapter 8. The authors build a two country, two sector model where

[3] Production relies on the presence of monopolistically produced intermediates, so there is also a subsidy to correct for the monopoly distortion.

one sector is differentiated and features productivity growth with local knowledge. The authors show that a country with a comparative advantage in that sector will keep that comparative advantage forever in laissez-faire, and, as in this paper, temporary research subsidies can allow the other country to catch up and eventually take the lead.[4]

The economic literature on trade and the environment has long recognized that in an open world, the effectiveness of unilateral policies in reducing world pollution can be hampered by the pollution haven effect - see for instance Pethig (1976), and empirical evidence can be found in Copeland and Taylor (2004). Markusen (1975) and Hoel (1996) show that the optimal instrument to address the pollution haven effect is a tariff. In the specific context of global warming, where the pollutant (CO_2) enters differently at several stages of the production process, several papers using computable general equilibrium models have attempted to track carbon through the global economy in order to determine the pattern of trade and compute the carbon leakage rate (that is the rate at which emissions abroad increase following a domestic reduction). It is generally agreed that the carbon contents of exports from developing countries to developed countries largely exceeds the carbon content of their imports.[5] Elliott et al. (2010) compute a carbon leakage rate of 20% from a reduction in Annex I countries (the countries with binding constraints under the Kyoto protocol), and show that border tax adjustments eliminate half of it.[6] The paper also relates to the literature on the impact of trade on the environment (see Copeland and Taylor (1995)): here in the absence of global cooperation, trade is necessary to avert an environmental disaster, but needs to be managed in order to deliver the right outcome.[7] This literature has focused on static models and has ignored the evolution of comparative advantage over time.

A growing literature has shown the importance of taking into account directed technical change when designing policies against global warming. On the empirical side, Popp (2002) shows that an increase in energy prices leads to more energy-saving innovation and Newell,

[4] Models featuring trade and directed technical change also include Acemoglu (2003), who studies the impact of trade on the skill bias of technological change, and Gancia and Bonfiglioli (2008), who show that trade amplifies international wage differences.

[5] For instance, Atkinson et al. (2011) find that the net US imports of carbon from China in 2004 were of 244 millions of tons of CO_2, which represents 0.9% of total world emissions that year.

[6] Other numerical studies delivering similar results include: Babiker and Rutherford (2005), Schenker and Bucher (2010), Böhringer, Fischer and Rosendahl (2010) or Böhringer, Carbon and Rutherford (2011). Introducing imperfect competition, Babiker (2005) finds a leakage rate greater than 100%, while, introducing the possibility for energy-saving innovation and international knowledge spillovers Gerlagh and Kuik (2007) find a negative rate. There are, however, few empirical studies: Aichele and Felbermayr (2010) use a gravity model of trade and find that committing to the Kyoto protocol increases the carbon content of imports from not-committed countries by 10%, while Aichele and Felbermayr (2012) find that when committing to the Kyoto protoc countries reduce domestic CO_2 emissions on average by about 7% but that their total CO_2 consumption does not change.

[7] Empirical studies (Antweiler, Copeland and Taylor (2001) or Frankel and Rose (2005)) point towards a positive effect of trade on the environment for local pollutents but the effect for CO_2 emissions is not clear.

Jaffe and Stavins (1999) find similar results in the air conditioner industry. Aghion et al. (2011) focus on the car industry and establish both that an increase in fuel prices leads to clean innovation at the expense of dirty innovation, and that there is path dependence in clean versus dirty innovation as in this paper. Following this literature, several theoretical papers have integrated directed technical change in the study of climate change policies; here, I build specifically on the model developed by Acemoglu et al. (2012) (henceforth AABH).[8] The final good in AABH and the polluting sector in my paper are both produced with a clean and a dirty input, which are substitutes for each other. Because of knowledge externalities of "the building on the shoulder of giants" type, there is path dependence in the direction of innovation (clean or dirty). AABH focus primarily on the single country case but the working paper already presents an international version of the model. Trade is between the clean and dirty inputs (which are substitutes), while in my current paper, it is between two sectors which are complements or Cobb-Douglas in consumption. In my model, innovation can reduce the emission rate in the polluting sector, so that a country can build a comparative advantage in the polluting sector while reducing its emissions; this is impossible in AABH. Moreover, AABH rule out innovation in the South. Maria and Smulders (2004) and Maria and van der Werf (2008) have also tackled the issue of modeling the interaction between directed technical change and international trade. These models study the allocation of innovation between an energy intensive sector and a non-energy intensive sector, but ignore that within the energy intensive sector innovations could be pollution-enhancing or saving.[9]

This paper is structured as follows: Section 2 presents the model, section 3 studies the laissez-faire equilibrium and which policies are able to ensure sustainable growth, section 4 solves for the optimal policy in the first best and in the second best when the South is constrained to be in laissez-faire, section 5 presents a stylized calibration, and finally, section 6 discusses how the main results are generalized when knowledge flows across countries. Appendix A presents some extensions of the model, Appendix B contains the main proofs, Appendix C gives details on the calibration and Appendix D contains additional proofs, the last two are available online.

[8] Earlier work on the environment and directed technical change include Bovenberg and Smulders (1995), Bovenberg and Smulders (1996), Goulder and Schneider (1999), van der Zwaan et al. (2002), Popp (2004), Grimaud and Rouge (2008) and Aghion and Howitt (2009).

[9] In Maria and Smulders (2004) the North develops technologies that the South imitates, and opening up to trade leads to a reallocation of innovation towards the sector that the North exports. Carbon leakage is reduced when the goods are substitutes and amplified otherwise. In Maria and van der Werf (2008) both countries innovate and carbon leakage is always reduced by the innovation response to a cut in emissions in a single country. Golombek and Hoel (2004) study the interaction between environmental policy and innovation in an open world in a static model.

2 Model

I consider an infinite-horizon version of a 2 country (North, N, and South, S), 2 sector (G and H), 2 + 1 factor (capital and labor plus scientists) Heckscher-Ohlin-Ricardo model, where sector G is similar to the economy of AABH. Time is discrete. Each country is endowed with a fixed amount of labor and capital: L_N, K_N and L_S, K_S, and there is a fixed mass one of scientists in both countries.

Welfare. I consider two distinct problems. In the first one, the economy admits for each period t a one-period lived representative agents in the North and one in the South.[10] The utility of time-t agent in country $X \in \{N, S\}$ is given by $\nu(S_t) C_{Xt}$ where S_t is the quality of the environment - identical in the North and in the South - and C_{Xt} is the final good consumption in country X. These preferences are aggregated under the social welfare function:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{(v(S_t)(C_{Nt} + C_{St}))^{1-\eta}}{1-\eta}. \tag{1}$$

where $\rho > 0$ is the discount rate, and $\eta \geq 0$ is the inverse elasticity of intertemporal substitution ($\eta = 1$ corresponds to a logarithmic utility). In this problem, the social planner will care only about the time profile of world consumption and the quality of the environment.

In the second problem, the economy admits an infinitely-lived representative agent in the North and one in the South, whose utility is given by $\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{(v(S_t)C_{Xt})^{1-\eta}}{1-\eta}$ and the social planner maximizes a weighted sum of these utilities:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{v(S_t)^{1-\eta}}{1-\eta} \left(\Psi C_{Nt}^{1-\eta} + (1-\Psi) C_{St}^{1-\eta}\right), \tag{2}$$

where $\Psi \in [0, 1]$ is the weight on the North's representative agent. In this case, the social planner also cares about the distribution of consumption across the two countries.

For both problems, ν is increasing in environmental quality S_t, C_{Xt} and S_t are weakly positive. There is an upper-bar on S_t denoted $\overline{S}$ which corresponds to a pristine environment. I will refer to an "environmental disaster" as a situation where environmental quality reaches in 0 in finite time. I further assume that $v(0) = 0$ and $v'(\overline{S}) = 0$ so that a disaster is as detrimental to welfare as zero consumption and the marginal damage of the first unit of pollution is zero.[11]

Production. Final consumption is a CES aggregate of the consumption of two goods denoted G and H:

$$C_{Xt} = \left(\nu C_{XGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu) C_{XHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

[10] As will be specified below, only the social planner will make an intertemporal decision.

[11] When a disaster occurs the economy is on an unsustainable path since the utility flow cannot be bounded away from the utility flow given by 0 consumption.

where C_{XYt} represents the quantity of good $Y \in \{G, H\}$ consumed in country $X \in \{N, S\}$. σ is the elasticity of substitution between goods G and H. I restrict attention to the case where the two goods are either complements ($\sigma < 1$) or where final consumption is Cobb-Douglas ($\sigma = 1$), so that both goods are essential for final consumption. Goods G and H will be the only goods traded internationally. Good G represents the traded goods responsible for greenhouse gases emissions (in particular, energy intensive goods), and good H, the traded goods which do not generate emissions. In the calibration section, good G will be identified with manufacturing of chemicals and chemical products (ISIC code 24), of other non-metallic mineral products (26) and of basic metals (27), and good H will be the rest of manufacturing.

Good H in country X is produced competitively according to

$$Y_{XHt} = \left(\int_0^1 A_{XHit} x_{XHit}^{\gamma} di \right) \left(K_{Xht}^{\beta} L_{Xht}^{1-\beta} \right)^{1-\gamma}, \tag{4}$$

where K_{Xht} (respectively L_{Xht}) is the capital (respectively labor) hired in the assembly of good H in country X, x_{XHit} is the quantity of intermediates i hired in sector H, and A_{XHit} is the productivity of intermediate i, specific to the country and the sector. γ represents the factor share of intermediates. Intermediates are produced monopolistically according to

$$x_{XHit} = \psi K_{XHit}^{\beta} L_{XHit}^{1-\beta}, \tag{5}$$

where K_{XHit} (respectively L_{XHit}) is the capital (respectively labor) hired in the production of intermediate i for good H in country X. Intermediates cannot be traded internationally. As the same factor share is used in the production of intermediates and in the final assembly of the good, $\beta \in (0, 1)$ is the overall factor share of capital in sector H.[12] I denote K_{XHt} total employment of capital in sector H in country X, so that:

$$K_{XHt} \equiv K_{Xht} + \int_0^1 K_{XHit} di, \tag{6}$$

similarly L_{XHt} is total employment of labor in sector H in country X.

Good G is produced competitively with a "clean" input Y_{Xct} and a "dirty" input Y_{Xdt} - which will be the sole source of pollution - according to:

$$Y_{XGt} = \left(Y_{Xct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{Xdt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where $\varepsilon > 1$ is the elasticity of substitution between the clean and the dirty input (I will also mention the perfect substitute case which corresponds to $\varepsilon = \infty$). The dirty input is the source

[12] The Cobb-Douglas structure of production for intermediates will ensure that monopolists get a constant share of the revenues generated by the sector, which matters for the incentives to innovate. Yet, the analysis can be extended straightforwardly to production functions where the repartition between capital and labor is not Cobb-Douglas.

of pollution. Both inputs are produced competitively according to:

$$Y_{Xzt} = \left(\int_0^1 A_{Xzit} x_{Xzit}^{\gamma} di\right) \left(K_{Xzt}^{\alpha} L_{Xzt}^{1-\alpha}\right)^{1-\gamma} \text{ for } z \in \{c, d\}, \tag{8}$$

where K_{Xzt} (respectively L_{Xzt}) is the capital (respectively labor) hired in the assembly of input z in country X, x_{Xzit} is the quantity of intermediates i hired in sector z, and A_{Xzit} is the productivity of intermediate i. Both clean and dirty intermediates are produced by monopolists according to:

$$x_{Xzit} = \psi K_{Xzit}^{\alpha} L_{Xzit}^{1-\alpha}, \tag{9}$$

so that $\alpha \in (0, 1)$ represents the total factor share of capital in sector G. The share of intermediates γ is the same for the clean input, the dirty input and sector H (so that the monopoly distortion only has a scale effect and does not affect the pattern of comparative advantage). I assume throughout that $\alpha > \beta$, which is true empirically: the most polluting sectors tend to be more capital intensive. This is without loss of generality, everything is identical when $\alpha < \beta$, and the analysis can be extended to a pure Ricardian model with $\alpha = \beta$.[13] I also define K_{XGt} as the total employment of capital in sector G:

$$K_{XGt} \equiv K_{Xct} + K_{Xdt} + \int_0^1 K_{Xcit} di + \int_0^1 K_{Xdit} di, \tag{10}$$

and define similarly L_{XGt} the total employment of labor in sector G in country X. In practice the clean input models non-polluting inputs that could substitute for polluting inputs, for instance renewable energies to replace fossil fuel energy, or bioplastics to replace traditional petroleum products (the functional form is discussed in greater extent in AABH).[14]

Market clearing for each factor in each country requires that:

$$K_{XGt} + K_{XHt} \leq K_X \text{ and } L_{XGt} + L_{XHt} \leq L_X, \tag{11}$$

and market clearing for each good requires that:

$$C_{NGt} + C_{SGt} \leq Y_{NGt} + Y_{SGt} \text{ and } C_{NHt} + C_{SHt} \leq Y_{NHt} + Y_{SHt}. \tag{12}$$

Innovation. At the beginning of every period, entrepreneurs receive monopoly rights on the production of intermediates such that each entrepreneur holds monopoly rights on the production of a finite number of intermediates for one period only. Moreover, entrepreneurs can hire scientists to increase the productivity of their variety. By hiring s_{Xzit} scientists, the

[13] The only issue in that case is that when the initial difference in comparative advantages is too small, it will be impossible to rule out multiple equilibria with different patterns of comparative advantages over time.

[14] I could have alternatively assumed that good G was produced with capital, labor and a CES aggregate of clean and dirty intermediates without affecting the results qualitatively for a sufficiently large elasticity of substitution ε.

entrepreneur holding the monopoly right on variety i in sector $z = H$ or subsectors $z \in \{c, d\}$ can increase the initial productivity $A_{Xi(t-1)}$ of his intermediate to:

$$A_{Xzit} = \left(1 + \kappa s_{Xzit}^{\iota} \left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}}\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma} A_{Xzi(t-1)}, \text{ for } z \in \{c, d, H\}, \tag{13}$$

where $0 < \iota < 1$, so that the innovation function κs^{ι} is increasing, concave and satisfies the Inada conditions (the analysis could be generalized to innovation functions of the form $\kappa\left((s_{Xzit} + \Upsilon)^{\iota} - \Upsilon^{\iota}\right)\left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}}\right)^{\frac{1}{1-\gamma}}$, with $\Upsilon > 0$, which do not satisfy Inada condition, such generalization will be useful in section 6). The concavity of the innovation function represents decreasing return to scale in innovation during a single period (the more scientists innovate on a particular technology during one period, the more they may reproduce the same type of innovation). A_{Xzt} is the average productivity of (sub)sector $z \in \{c, d, H\}$, at time t, defined as:

$$A_{Xzt} \equiv \left(\int_0^1 A_{Xzit}^{\frac{1}{1-\gamma}} di\right)^{1-\gamma} \text{ for } z \in \{c, d, H\}. \tag{14}$$

The factor $A_{Xzi(t-1)}^{-\frac{1}{1-\gamma}}$ represents decreasing return to scale in innovation (the more advanced is a technology the more difficult it is to further innovate), $A_{Xz(t-1)}^{\frac{1}{1-\gamma}}$ represent knowledge spillovers from all the other intermediates in the same sector in the same country. These two last effects exactly compensate each other and do not create any inefficiency in the economy. This formulation ensures that the innovation decision remains symmetric across varieties and that aggregate productivity grows exponentially for a given mass of scientists working in the (sub)sector. In both countries, there is a mass 1 of scientists, and market clearing requires that:

$$\int_0^1 (s_{XHit} + s_{Xcit} + s_{Xdit})\, di \leq 1. \tag{15}$$

Since entrepreneurs have monopoly rights for one period only, they will hire scientists so as to maximize their current profits instead of the entire flow of profits generated by their innovation. The allocation of scientists across (subsectors) is therefore "short-sighted". One period monopoly rights are the only inefficiency in innovation and allow to model in the simplest way the "building on the shoulder of giants" externality. The existence of this externality has long been recognized by the endogenous growth literature. In the specific context of climate change it plays a crucial role in explaining why clean technologies have failed to really take off so far, and why direct research incentives on top of carbon taxes are welfare improving (this is the point made by AABH). With permanent monopoly rights, infinitely-lived agents and no environmental externality, the efficient innovation allocation would be an equilibrium but in general not the unique one.

Further, note that there are no technological spillovers between countries and that technologies are country specific. Section 6 analyzes alternative scenarios. A fixed mass of scientists in both countries allows to focus on the direction of technical change only, and an equal mass in both countries ensures that one country does not become arbitrarily large relative to the other one (this assumption is relaxed in Appendix A).

Finally, it is important that innovation can occur in all three (sub)sectors. If innovation were limited to clean and dirty technologies within the polluting sector, the North could not build a comparative advantage in a specific sector which will be crucial here. With clean innovation in the polluting sector only, the model would ignore all innovations that are directed at increasing productivity without decreasing emissions - to some extent this is what Maria and Smulders (2004) and Maria and van der Werf (2008) do - while, in fact, this is still the bulk of innovation in manufacturing (Aghion et al. (2011) show that in the car industry there are more "dirty" innovations than "clean" ones). If, on the contrary, only dirty innovations were available in the polluting sector, it would not be possible to generate innovation to "replace" existing polluting technologies, as the polluting sector and the non-polluting sector are complements ($\sigma \leq 1$). The working paper version of AABH deals with this case, when the two sectors are substitutes ($\sigma > 1$).

Environment. Within the two bounds 0 and $\overline{S}$, environmental quality evolves according to:

$$S_t = \Delta \overline{S} + (1-\Delta) S_{t-1} - \xi \left(Y_{dNt} + Y_{dSt}\right) \text{ for } S_{t-1} > 0, \tag{16}$$

$$S_t = 0 \text{ for } S_{t-1} = 0. \tag{17}$$

This assumes that the disaster level is an absorbing state - which represents the notion that for sufficiently high greenhouse gases concentration, the climate may move towards a different long-run equilibrium. $\xi > 0$ measures the rate of environmental degradation from the production of dirty input, and $\Delta > 0$ is the regeneration rate of the environment. Without loss of generality, I assume that $S_0 = \overline{S}$. This representation matters for the calibration but it is adopted in the analytical part for simplicity's sake only. The analytical results can easily be generalized to different laws of motion: the only assumptions used are that environment regenerative capacity is limited and, for some results on the optimal policy only, that the disaster level is an absorbing state.

Policy tools. Section 4 will solve the social planner's problem of maximizing (1) or (2), but the following section only studies whether an environmental disaster can be prevented or not with some specific policy instruments. The instruments that I will focus on are ones that can decentralize the optimal solution and correspond to the policies that are implemented or considered for implementation. More specifically, I will introduce add-valorem taxes on the dirty input (τ_{Xt}), which are the equivalents of a carbon tax, and sector specific add-valorem research subsidies or taxes on scientists' wages - a country may also implement a subsidy to the use of all intermediates identical across every subsector to correct for the monopoly distortion.[15] In addition, when I look at unilateral policies for the North, I allow for an add-valorem trade tax on the polluting good G (because of Lerner symmetry this is without loss of generality, the trade tax could also be on the other good). Therefore, prices in the South are always equal to international prices: $p_{SGt} = p_{Gt}$ and $p_{SHt} = p_{Ht}$, while in the North the price of good H is also equal to the international price $p_{Ht} = p_{NHt}$, but the price of good G is given by $p_{NGt} = p_{Gt}(1 + b_t)$, where b_t is the trade tax. A positive trade tax corresponds to a tariff when the North imports good G and to an export subsidy when it exports it.[16] When the North is the only country intervening, I assume that trade balance must be maintained (there is no intertemporal trade), where trade balance writes as:

$$p_{Gt}(Y_{SGt} - C_{SGt}) + p_{Ht}(Y_{SHt} - C_{SHt}) = 0. \tag{18}$$

Note that the trade tax is not explicitly related to the carbon content of imports. When the South does not undertake any policy, relating it explicitly to the average carbon content of imports from a given country and in a given sector, would not change anything (each Southern firm being atomistic, its impact on average emission is infinitesimal and so its behavior will not affect the trade tax it pays). To affect the behavior of Southern firms, the North would either need to know the carbon content of its imports individually for each exporting firm - which seems implausible - or the South would have to implement a policy in response to the North tariff. I come back to these issues in subsection 3.4. Overall, a policy is characterized by a sequence of add-valorem taxes on the dirty input τ_{Xt} in each country, a sequence of taxes and subsidies on scientists in every subsector, and a sequence of trade taxes b_t on the polluting good. All subsidies and taxes are financed (or rebated) through lump-sum taxation at the country-level.

3 Preventing an environmental disaster

This section studies what type of policies can prevent the environmental quality from reaching zero (that is can prevent an environmental disaster). In the first subsection, I detail the

[15] In order to ensure uniqueness of the equilibrium allocation of scientists, I assume that it is possible to give the subsidy to only a given mass of scientists, so that the social planner can use it to determine the exact allocation. Note that if the subsidy is greater than 100%, monopolists may be willing to hire scientists even if they are not producing any good. This assumption clarifies the exposition, I discuss in the paper how the results would be affected if this was impossible.

[16] Starting from a situation where in free-trade the North imports the polluting good, a positive trade tax will correspond to a tariff up to the point where it reproduces autarky. Beyond that point, the North would start exporting the polluting good and the positive trade tax corresponds to an export subsidy.

behavior of the economy under laissez-faire; in particular, I explain the pattern of trade and the allocation of innovation across sectors. In subsection 3.2, I analyze how sustainable growth can be achieved with policy in both countries, and subsection 3.3 explains why taxing the polluting sector in the North only can fail at preventing a disaster. Subsection 3.4 explains how a disaster can be avoided with unilateral policies in the North, subsection 3.5 discusses some extensions and subsection 3.6 summarizes the results. For a given policy, the equilibrium is defined as follows.

Definition 1 *A feasible allocation is a sequence of demands for capital* (K_{Xht}, K_{XHit}, K_{Xct}, K_{Xcit}, K_{Xdt}, K_{Xdit})*, demands for labor* (L_{Xht}, L_{XHit}, L_{Xct}, L_{Xcit}, L_{Xdt}, L_{Xdit})*, demands for intermediates* (x_{Xzit} *for* $z \in \{c, d\}, H$)*, demands for inputs* (Y_{Xct}, Y_{Xdt})*, goods production* (Y_{XGt}, Y_{XHt})*, demands for goods* (C_{XGt}, C_{XHt})*, research allocations* (s_{Xzit} *for* $z \in \{c, d\}, H$) *and quality of the environment* S_t*, such that in each period* t *and in each country* $X \in \{N, S\}$*, factor and good markets clear ((11), (15), and (12) hold).*

Definition 2 *For a given policy, an equilibrium is given by a feasible allocation and sequences of wages of workers* (w_{Xt})*, returns to capital* (r_{Xt})*, wages of scientists* (v_{Xt})*, (consumer) prices for intermediates* (φ_{Xzit} *for* $z \in \{c, d\}, H$)*, (producer) prices for clean and dirty inputs* (p_{Xct}, p_{Xdt})*, international prices of goods* (p_{Gt}, p_{Ht}) *for* $X \in \{N, S\}$*, such that (i)* ($\varphi_{Xzit}, x_{Xzit}, s_{Xzit}, K_{Xzit}, L_{Xzit}$) *maximizes profits by the producer of intermediate* i *in sector* $z \in \{c, d, H\}$ *in country* X*, (ii)* L_{Xzt}, K_{Xzt} *maximize the profits of the producer of good* $z \in \{c, d, H\}$*, (iii)* Y_{Xct} *and* Y_{Xdt} *maximize the profits of producer of good* G*, (iv)* C_{XGt} *and* C_{XHt} *maximize consumers' utility under the trade balance constraint, (v) (18) is satisfied.*

3.1 Laissez-faire

Trade pattern. This subsection analyzes the laissez-faire equilibrium, the results are derived and generalized in Appendix B.1. First, in each country aggregate production in each sector can be written as:

$$Y_{XGt} = \zeta A_{XGt} K_{XGt}^{\alpha} L_{XGt}^{1-\alpha} \text{ and } Y_{XHt} = \zeta A_{XHt} K_{XHt}^{\beta} L_{XHt}^{1-\beta}, \tag{19}$$

with $\zeta \equiv \frac{\gamma^{2\gamma}(1-\gamma)^{1-\gamma}}{(1-\gamma+\gamma^2)\psi^{\gamma}}$ and $A_{XGt} \equiv \left(A_{Xct}^{\varepsilon-1} + A_{Xdt}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$ is the average productivity of sector G when there is no carbon tax. This formulation highlights that, in a given period, the model collapses into a Heckscher-Ohlin model with varying productivity across countries. The South has the comparative advantage in the polluting good G and exports it if and only if

$$\left(\frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NGt}}{A_{NHt}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}. \tag{20}$$

Trade results both from Ricardian forces (relative productivity) and Heckscher-Ohlin forces (relative endowment). As long as the difference in comparative advantage is not too large, both countries will produce both goods, but once it becomes sufficiently large one country fully specializes and, if the difference in comparative advantage grow even further, both countries fully specialize. Emissions are given by: $E_{Xt} = \xi \left(\frac{A_{Xdt}}{A_{XGt}}\right)^{\varepsilon} Y_{XGt}$, so that the emission rate in the polluting sector increases in the relative productivity of the dirty technology A_{Xdt}/A_{Xct}. Over time, innovation will modify comparative advantage and the emission rate.

Allocation of innovation. The innovation decision results from two forces: path dependence in clean versus dirty technologies and amplification of comparative advantage. Entrepreneurs face a two stages problem. In the second stage, they choose prices in order to maximize their profits given their productivity. Post-innovation profits in sector $z \in \{c, d, H\}$ are given by (see Appendix B.2):

$$\pi_{Xzit} = (1-\gamma)\gamma \left(\frac{A_{Xzit}}{A_{Xzt}}\right)^{\frac{1}{1-\gamma}} p_{Xzt} Y_{Xzt}. \tag{21}$$

These profits are directly proportional to the revenues of their (sub)sector and increase with the productivity of their intermediate A_{Xzit}. In the first stage, entrepreneurs hire scientists to increase the productivity of their intermediate. Thanks to the knowledge spillovers across varieties, all monopolists in a given (sub)sector hire the same number of scientists, so that average productivity evolves according to:

$$A_{Xzt} = \left(1 + \kappa s_{Xzt}^{\iota}\right)^{1-\gamma} A_{Xz(t-1)} \text{ for } z \in \{c, d, H\}.$$

Path dependence in clean versus dirty technologies. Assume that country X produces some good G (otherwise $s_{Xct} = s_{Xdt} = 0$). Combining the first order conditions with respect to the number of scientists in the clean and dirty subsector (and assuming that some production takes place in sector G in country X), the allocation of scientists within sector G obeys:

$$\frac{s_{Xct}^{1-\iota}\left(1 + \kappa s_{Xct}^{\iota}\right)}{s_{Xdt}^{1-\iota}\left(1 + \kappa s_{Xdt}^{\iota}\right)} = \frac{p_{Xct} Y_{Xct}}{p_{Xdt} Y_{Xdt}} = \frac{A_{Xct}^{\varepsilon-1}}{A_{Xdt}^{\varepsilon-1}}. \tag{22}$$

The second equality arises from the demand equation for both inputs in sector G and the fact that their production technologies differ only by their level of productivity. The ratio of revenues in the clean over the dirty sector increases with the ratio of clean over dirty technologies. A larger average productivity leads to a larger market share for the input (so if $A_{Xdt} > A_{Xct}$, $Y_{Xdt} > Y_{Xct}$), it also makes the input more expensive ($p_{Xdt} < p_{Xct}$), but when the inputs are substitutes, this price effect is dominated by the market size effect. Thus, more scientists are allocated to the dirty subsector than to the clean subsector if and only if $A_{Xd(t-1)} > A_{Xc(t-1)}$, as long as the size of innovation κ is sufficiently small (otherwise there may be multiple equilibria when $A_{Xd(t-1)}$ and $A_{Xc(t-1)}$ are close to each other, see Appendix B.2). Hence, in the polluting sector G, in the absence of government intervention, innovation tends to be allocated to the sector already the most advanced: there is path-dependence.

Amplification of comparative advantage. Assume that production holds in both sectors (otherwise innovation only occurs in the active sector). Combining the first order condition with respect to the number of scientists in sector H and in subsectors c and d, I get:

$$\frac{s_{Xct}^{1-\iota}\left(1+\kappa s_{Xct}^{\iota}\right)+s_{Xdt}^{1-\iota}\left(1+\kappa s_{Xdt}^{\iota}\right)}{s_{XHt}^{1-\iota}\left(1+\kappa s_{XHt}^{\iota}\right)}=\frac{p_{XGt}Y_{XGt}}{p_{XHt}Y_{XHt}}. \tag{23}$$

Therefore, for given ratio of initial productivities within sector G ($A_{Xd(t-1)}/A_{Xc(t-1)}$ given), the larger the revenues in sector G relative to sector H, the more scientists will be allocated to sector G.

In autarky, using consumer demand:

$$\frac{p_{XHt}Y_{XHt}}{p_{XGt}Y_{XGt}}=\frac{1-\nu}{\nu}\left(\frac{Y_{XG}}{Y_{XH}}\right)^{\frac{1-\sigma}{\sigma}}, \tag{24}$$

so that if $\sigma=1$, innovation remains balanced between the two sectors as the right-hand side term is a constant, while for $\sigma<1$, innovation tends to occur in the smallest sector – that is the sector with relatively lower productivity – and therefore becomes balanced between the two sectors after a few periods where the laggard sector catches up. Since the two sectors are complements, innovation will not disappear in one sector over time (as it does in the case of clean versus dirty innovation).

Under free-trade, prices are equalized in the North and the South, so each country tends to innovate relatively more in the sector it exports (and does so at equal ratio of initial productivities within sector G). As more innovation in a sector leads to a larger comparative advantage in that sector, which typically prompts more innovation in the same sector in the first place, multiple equilibria could arise. One can however show the following:

Lemma 1 *If κ is small enough and $\iota \geq 1/2$ the equilibrium is unique.*

Proof. See Appendix D.1 ■

From now on, I will assume that these conditions are satisfied so that the equilibrium is unique. A sufficiently small size of innovation κ ensures changes in productivities in one period remain sufficiently small, while the technical assumption $\iota \geq 1/2$ is necessary to ensure that an equilibrium with full specialization does not coexist with one without full specialization.[17] I can then derive:

[17] Having $\iota < 1/2$ would, however, not affect the any other result in this section, nor in section 6. The results of section 4 would also hold as long as an interior equilibrium is chosen whenever it exists.

Lemma 2 *In laissez-faire, if the South initially has a weak comparative advantage in sector G* $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} \geq \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ *and* $\min\left(A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0}\right) < \min\left(A_{Nc0}/A_{Nd0}, A_{Nd0}/A_{Nc0}\right)$, *then every period more scientists are hired in sector G in the South than in the North:* $s_{SGt} > s_{NGt}$*; and, the South fully specializes in producing good G and the North in producing good H in finite time.*[18]

Proof. See Appendix B.2 ■

Because countries tend to innovate more in the sector they export, the difference in the allocation of research across sectors builds up over time and the relative productivities of both sectors become so different that the two countries eventually fully specialize. The condition on $\min\left(A_{Xc0}/A_{Xd0}, A_{Xd0}/A_{Xc0}\right)$ is necessary for the following reason: The incentive to innovate in sector G and the growth rate of the average productivity of sector G for a given mass of scientists in that sector depend on the initial relative productivities in clean and dirty technologies $A_{Xc(t-1)}/A_{Xd(t-1)}$ (both decrease when $A_{Xc(t-1)}/A_{Xd(t-1)}$ is close to 1); with the assumption of the lemma, this aspect does not prevent innovation from going to the sector of initial comparative advantage. This increase in comparative advantage effect is where the mechanics of the model bear some resemblance with the infant industry argument, as initial comparative advantage affects the path that the economy undertakes.

3.2 Avoiding a disaster with policy in both countries

If the dirty subsector is more advanced than the clean one in both countries, then under laissez-faire, innovation in sector G is continually directed primarily towards dirty intermediates. As innovation necessarily keeps occurring in both sectors,[19] the production of good G and emissions grow unboundedly. At some point the regeneration capacity of the environment becomes overwhelmed and the economy reaches a disaster.

Using clean research subsidies, taxes on dirty research or a carbon tax, a global government could redirect innovation from the dirty towards the clean subsector in both countries. Once clean technologies will have acquired a sufficient lead over dirty intermediates, market forces will ensure that research is continually directed mostly towards the clean subsector (now the most advanced). Eventually, the emission rate of the polluting good goes towards 0, and a disaster can be avoided if the initial environmental quality is large enough. This analysis is

[18] The lemma extends to the case where $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and $\min\left(A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0}\right)$ is sufficiently small. If $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} = \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and $\min\left(A_{Sc0}/A_{Sd0}, A_{Sd0}/A_{Sc0}\right) = \min\left(A_{Nc0}/A_{Nd0}, A_{Nd0}/A_{Nc0}\right)$, there is no trade.

[19] The exporting country innovates more in the polluting good than it would under autarky and, since goods G and H are either complements or Cobb-Douglas, innovation does not disappear within the polluting good under autarky.

similar to AABH and is presented here to provide contrast with the unilateral policy case. It can be summarized in the following remark:

Remark 1 No matter how large $\overline{S}$ is a disaster occurs in the laissez-faire equilibrium if clean technologies are less developed than dirty ones ($A_{Nc0} \leq A_{Nd0}$ and $A_{Sc0} \leq A_{Sd0}$). For sufficiently large $\overline{S}$, temporary clean research subsidies, taxes on dirty research, or carbon taxes in both countries can prevent a disaster.

Proof. See Appendix D.2 ■

3.3 Taxes on the polluting good in the North only

Assume now that only the North can implement some policy. Can it avoid an environmental disaster alone? First note that in autarky and without knowledge spillovers, no policy in the North could prevent a disaster, as emissions in the South alone would grow unboundedly regardless of what the North does: trade is necessary to avoid an environmental disaster without international cooperation. Now one can show:

Lemma 3 *If clean technologies are less developed than dirty ones in the South $A_{Sc0}/A_{Sd0} \leq 1$, then, to prevent a disaster, all factors in the South must be asymptotically allocated to the non-polluting sector H, and the North must export the polluting good G in the long-run.*

Proof. See Appendix B.3 ■

In other words, the key to avoid an environmental disaster with policies in the North only is to ensure that the South economy asymptotically fully specialize in the non-polluting sector. If the South were to keep allocating a positive share of labor and capital to the polluting sector, the mass of scientists allocated to the polluting sector in the South will never go to 0 either (since the two goods are complements). The production of polluting good in the South would become unbounded, and without an environmental policy in the South, so will emissions.

In this subsection, I focus on taxes on the polluting good in the North (a carbon tax or a tax on dirty research), which are non-protectionist policies.[20] Both can reduce emissions in the North and prompt clean innovation there, and could prevent an environmental disaster if the North were alone or if the South were to undertake the same policy, but such policies may be incompatible with a South specializing in the non-polluting sector and therefore may be unable to prevent an environmental disaster.

Proposition 1 *No matter how large $\overline{S}$ is, no combination of a carbon tax and a tax on dirty research can prevent a disaster if clean technologies are less developed than dirty ones in the North*

[20] The equilibrium is still unique with these instruments if the tax on dirty research can be restricted to apply only to a fraction of scientists.

$A_{Nc0}/A_{Nd0} \leq 1$, *clean technologies are sufficiently less developed than dirty ones in the South (A_{Sc0}/A_{Sd0} is sufficiently small and in particular $A_{Nc0}/A_{Nd0} \geq A_{Sc0}/A_{Sd0}$), and the South has a weak initial comparative advantage in the polluting sector* $(A_{SG0}/A_{SH0})^{\frac{1}{\alpha-\beta}} K_S/L_S \geq (A_{NG0}/A_{NH0})^{\frac{1}{\alpha-\beta}} K_N/L_N$.

Proof. See Appendix B.4 ■

This result follows lemmas 2 and 3. Under laissez-faire and with the assumptions of the proposition, the South would keep the comparative advantage in the polluting sector or (in the knife edge case) there would never be trade. Using a tax on dirty research or a carbon tax, the North government cannot reverse this pattern. On the contrary: a tax on dirty innovation drives scientists away from the polluting sector G towards the non-polluting sector H, and, within the polluting sector it allocates innovation towards the initially backward clean subsector, further reducing the growth rate of the average productivity A_{NGt}. A carbon tax has the same effect on innovation and also directly reduces the productivity of the polluting sector in the North. As both instruments increase the costs of producing the polluting good in the North, they lead to an increase in its world relative price. This induces an increase in the production of the polluting good G in the South and therefore more emissions in the South, which is the classic pollution haven effect. As the relative revenues of the polluting sector increase in the South, more innovation there takes place in the polluting sector, where it is mostly directed at the dirty technologies. North domestic taxes on the polluting good can therefore only accelerate the specialization of the South in the polluting sector.

In fact, the economy tends to grow faster when countries are more specialized as there is less overlap in the type of innovations that they are undertaking. As a result faster specialization will typically lead to faster economic growth and more emissions. In addition, since the gap between clean and dirty technologies in the South will grow faster, the emission rate will also grow faster. Therefore, it is likely that such policies lead to faster environmental degradation. Of particular interest is the knife edge case where the South has no comparative advantage ($(A_{SG0}/A_{SH0})^{\frac{1}{\alpha-\beta}} K_S/L_S = (A_{NG0}/A_{NH0})^{\frac{1}{\alpha-\beta}} K_N/L_N$ and $A_{Nc0}/A_{Nd0} = A_{Sc0}/A_{Sd0}$). In this case, there would be no trade in laissez-faire, but because of the pollution haven effect, the policy intervention tips the balance towards a comparative advantage for the South in sector G, which then builds on itself over time: there is more economic growth and the environmental disaster is reached sooner. Note also that under such policy, innovation within the polluting sector in the North is tilted towards clean technologies rather than dirty ones. However, since the production of the polluting sector is moving to the South, the market size for clean technologies in the North is too small for innovation there to increase sufficiently fast.

In the proposition, the condition that A_{Sc0}/A_{Sd0} must be sufficiently small (and not simply smaller than 1) is necessary for the same reasons as in lemma 2: with the ratio of clean over dirty technologies further from 1 in the North than in the South, more innovation in the polluting sector might take place in the North than in the South even if the South exports the polluting good.[21] Such condition can be dispensed with if the initial comparative advantage is sufficiently large.[22] If the North has the initial comparative advantage in the polluting sector G, the pollution haven effect still pushes the South towards specializing in the polluting sector, but the amplification of initial comparative advantage effect now pushes in the other direction.

3.4 Introducing clean research subsidies and trade taxes

The previous policies failed a preventing an environmental disaster when the South had the initial comparative advantage in the polluting sector because they could not reverse the pattern of trade. I now allow the North to use clean research subsidies and a trade tax.

Proposition 2 *A combination of a temporary trade tax and a temporary clean research subsidy in the North can prevent a disaster provided that the initial environmental quality* $\overline{S}$ *is sufficiently large.*

Proof. See Appendix B.5.1 ■

Clean research subsidies will be able to redirect innovation in the North from the dirty technologies and the non-polluting sector H towards clean technologies: as a result the emission rate in the North can be reduced without necessarily losing competitiveness relative to the South. The trade tax can be used to reduce specialization in the South and prevent too much innovation there in the polluting sector. As a result, for sufficiently large initial environmental quality, a combination of these two policies can prevent a disaster. For instance, the following two phases approach works. In a first phase, the social planner implements a tariff large enough to shut down trade, so that innovation in the South must be balanced between the polluting and non-polluting sectors. In the meantime, the social planer can implement large clean research subsidies so that nearly all scientists in the North innovate in the clean subsector. As a consequence, not only do clean technologies become more advanced than dirty ones in the North, but the North builds a comparative advantage in the polluting sector G, since

[21] More specifically, the incentive to innovate in sector G is lower when the revenues in the clean and dirty subsectors are close to each other, that is when $A_{Xc(t-1)}^{\varepsilon-1}$ and $(1+\tau_{Xt})^{-\varepsilon} A_{Xd(t-1)}^{\varepsilon-1}$ are close to each other. With a tax on dirty research only, it is possible to get $A_{Ndt}/A_{Nct} < A_{Sct}/A_{Sdt}$ at some point once clean technologies in the North have caught up with dirty ones, by preventing any research in dirty technologies. A sufficiently large carbon tax can make the dirty subsector arbitrarily less profitable than the clean one in the North, regardless of relative technologies. The assumption on A_{Sc0}/A_{Sd0} ensures that the difference in comparative advantages becomes sufficiently large should the revenues ratio in clean over dirty be further than one in the North than in the South.

[22] In the perfect subsitute case this condition would be replaced by $A_{Sc0}/A_{Sd0} < (1+\kappa)^{-(1-\gamma)}$ which ensures the unicity of the equilibrium in the South. In the pure Ricardian case $\alpha = \beta$, the proposition holds for sufficiently large initial comparative advantage.

it innovates relatively more in the polluting sector than the South does. Once the North has acquired the comparative advantage in the polluting sector and $A_{Nc(t-1)}/A_{Nd(t-1)}$ is sufficiently large, the social planner can stop all policies: following lemma 2, sector G production ends up fully moving to the North (where it has become clean), and a disaster can be avoided. Here note that the trade tax can simply take the form of a tariff to be implemented when the North imports the polluting good G (reproducing autarky is enough, it is not necessary to impose an export subsidy - see footnote 16).[23] Since clean research subsidies are able to completely revert the pattern of innovation in the North, one may think that this instrument alone could be enough to prevent an environmental disaster, however, I can show:

Remark 2 If final consumption is Cobb-Douglas in the polluting and non-polluting goods ($\sigma = 1$), there exist initial factor endowments and technologies such that no matter how large $\overline{S}$ is, no combination of a carbon tax, a tax on dirty research and a subsidy on clean research can prevent a disaster.

Proof. See Appendix B.5.2 ■

Therefore, clean research subsidies may still fall short in preventing an environmental disaster if enacted alone. This occurs when the South fully specializes in the polluting sector and clean technologies in the South are sufficiently less advanced than dirty ones. In this case all South scientists are allocated to the polluting sector and asymptotically all of them to dirty technologies. Therefore, even if the North were to allocate all its scientists to clean technologies, A_{SGt} would grow as fast as A_{NGt}. Such a situation is irreversible in the Cobb-Douglas case, and an environmental disaster cannot be avoided. When the two goods are strict complements ($\sigma < 1$), on the contrary, if both countries innovate in the polluting sector only, the demand for the non-polluting good becomes so large that the South cannot stay fully specialized.[24] Full specialization in the South will happen in the first place if its initial comparative advantage in the polluting sector is sufficiently large or if clean technologies are sufficiently backward in the North (as the average productivity of the polluting sector in the North A_{NGt} grows slowly during the period where clean technologies are catching up with dirty ones there).

3.5 Extensions

South's retaliation. So far, we have assumed that there is no government in the South who could retaliate to the unilateral policies imposed by the North. Although a full analysis

[23] In the presence of a trade tax, the equilibrium may not be unique when $\sigma < \frac{1}{2}$, but this does not affect the analysis.

[24] See Appendix D.3. Note, that averting a disaster with clean research subsidies in that case relies on our assumption that monopolists would be willing to hire scientists for sufficiently large subsidies even if they are not producing any good.

of the strategic interactions between two governments is beyond the scope of this paper, it is worth studying briefly the scope for South's intervention. First, note that the South is not necessarily going to lose consumption out of the unilateral policies enacted by the North - the South will always benefit from the better environmental quality. For instance, if the North implements a temporary policy to reverse the pattern of comparative advantage, both countries fully specialize in the long-run. In the Cobb-Douglas case ($\sigma = 1$), income shares are linked to the consumption share of the good that the country exports: by specializing in the non-polluting sector (with a share $1 - \nu$) instead of the polluting one (with a share ν), the South is likely to get a larger consumption share in the long-run since empirically one can expect $1 - \nu > \nu$. Even in the short-run, the South could benefit: although it is true that when the South exports the polluting good a tariff on it will hurt the South, the trade tax implemented by the North could be sufficiently large to reverse the pattern of trade, so that it is actually an export subsidy which benefits the South (this is in fact what happens in the calibration for the maximization of (1) - see subsection 5.5).

Second, one could consider the case of a Nash equilibrium between a North social planner who maximizes the welfare of an infinitely lived North representative agent, and a South social planner who maximizes the welfare of an infinitely lived South representative agent, with identical preferences. In this case, the equilibrium will not be at the first best, and environmental degradation is likely to be larger, however, an environmental disaster will still always be averted - as consumption does not increase welfare if environmental quality reaches 0.

Maybe a more interesting case to look at is one with a South's government which maximizes only current consumption. In that case, facing the unilateral policy imposed by the North, the South will implement its own trade tax to improve its terms of trade. In the first phase, when the South retains an initial comparative advantage in the polluting good, this trade tax will move both countries closer to autarky and therefore does not prevent the North from reversing the pattern of comparative advantage. In the second phase (when the North does not need to impose a trade tax any more) the South will implement a tariff on its imports of the polluting good. This tariff will slow down the specialization of the South in the non-polluting sector. However, one can show that if the North has acquired enough of comparative advantage, the South still eventually specializes in the non-polluting sector and a disaster can be avoided for sufficiently large initial environmental quality.[25]

Good-based carbon content tariff. As discussed in section 2, without policy in the

[25] This claim is established when North does not implement its own trade tax. Alternatively the North could undo any impact of a trade tax implemented by the South on the pattern of production and emissions by imposing the opposite trade tax (but this would come at the cost of a wealth transfer from the North to the South).

South, the only way for the North to directly affect Southern firms' emission rates is to know the carbon contents of each imports at the firm level, which, without any cooperation from the South, seems impossible. Nevertheless, it is interesting to mention this case as a benchmark (and it is more relevant when there are only few exporters in the South). In this case, when the South has the comparative advantage in the polluting good, the dirty input will be taxed for the exports market. If clean technologies are not too backward in the South, and the export market is relatively large (which is possible if the South is small and has a large comparative advantage), a switch to clean technologies in the South is possible, and a reversal of comparative advantage may not be necessary.[26] When the South is willing to undertake some form of policy on the other hand, a realistic approach could be to relate the carbon tariff with average emissions from a given country in a given sector. In this case, the South government would internalize that the tariff depends on its policy and could be incentivized to undertake some environmental policy on its own. Such an analysis would be very interesting but is beyond the scope of this paper. In Appendix A.1, I study the role of other instruments and the case where the mass of scientists is different in the North and in the South.

3.6 Taking stock and comparison with the no DTC case

The following lessons can be derived from the previous analysis. First, the pollution haven effect becomes worse in a dynamic setting. Taxes on the polluting sector in the North risk leading the economy to a path where the South has the comparative advantage in the polluting sector. As comparative advantage tends to get reinforced over time, the bulk of production of the polluting sector ends up occurring in the South. This dramatically hampers the impact of the intervention on worldwide emissions. Furthermore, note that since the market share for the polluting sector becomes small in the North, the incentives to innovate in clean technologies remain very limited. To ensure sustainable growth without cooperation from the South, the North must undertake a temporary industrial policy in order not only to make the polluting sector cleaner but also to get the comparative advantage in the polluting sector.

Second, trade acts as a double-edged sword. In laissez-faire, trade leads to specialization, which maximizes long-run growth and therefore leads to faster environmental degradation. Moreover, because of the pollution haven effect, trade makes non-protectionist policies less efficient. However, if well managed, trade is key to avoid a disaster in a non cooperative world, since once the North has developed its clean technologies sufficiently, trade forces ensure that pollution will decrease in the South too.

Third, directed technical change also acts as a double-edged sword. Relative to a model

[26] Note that such a trade tax makes sense as a tariff only, not as an export subsidy. A combination of clean research subsidies and a good-based carbon content tariff can prevent a disaster for sufficiently large environmental quality.

where technological levels grow exogenously at an equal rate, directed technical change accelerates economic growth and environmental degradation, and reduces the benefits from non-protectionist unilateral policies through the South innovation response. On the other hand, it allows a reversal of the pattern of comparative advantage which is key in preventing a disaster with unilateral policies. Crucially, for some parameters, avoiding a disaster with unilateral policies will be impossible without directed technical change. When clean technologies are sufficiently less developed than dirty in the North and the South has a large comparative advantage in the polluting sector, then even if the North produces only the polluting good in a clean way and exports all of its production to the South (which can be achieved with sufficiently large trade and carbon taxes), there will not be enough export to ensure that the South specializes in the non-polluting sector. Since, in this case, the pattern of comparative advantages does not change with time, the South will never specialize and emissions in the South will grow unboundedly. This shows that the ability to affect comparative advantages with innovation was key in deriving the previous results. I come back to the role of trade and directed technical change in the calibration section.

4 Optimal policy

I now analyze the optimal policy. I In this section, I briefly characterize the first best policy, which generalizes the analysis of AABH, and then I study the second best case where the social planner cannot intervene in the South.

4.1 First best

In the first best, the social planner maximizes (1) or (2) under the production function equations (3), (4), (5), (7), (8), (9), the factor market clearing equations (11), (15), the good market clearing equations (12), the environmental degradation equation (16) and the accumulation of knowledge equations (13). I can then show:

Proposition 3 *The first best policy can be decentralized through a combination of a carbon tax in the North and in the South (with the same price for carbon), research subsidies/taxes in the North and the South in both sectors and a subsidy to the use of all intermediates. When the social planner maximizes (2) international transfers are also required.*

Proof. See Appendix B.6 ■

Each instrument allows the social planner to correct for one distortion. First, the environmental externality is corrected by a carbon tax in both countries which equalizes the marginal cost of the tax in terms of lower consumption at the time, with the marginal benefit of higher environmental quality in all subsequent periods from avoiding one unit of pollution (carbon taxes in the North and the South differ in add-valorem form across countries but are identical as a per-unit of CO_2 tax). Second, the social planner corrects for the myopia of monopolists in their innovation decisions. Instead of allocating scientists across sectors depending on the revenues generated by their effort in their sector in the first period, the social planner allocates them according to the discounted value of the entire stream of additional revenues generated by their innovation. More specifically, instead of (22) and (23), scientists are now allocated across the dirty, clean and H (sub)sectors according to:

$$\frac{s_{XHt}^{\iota-1}}{1+\kappa s_{XHt}^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{Hs}Y_{Hs}=\frac{s_{Xct}^{\iota-1}}{1+\kappa s_{Xct}^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{Xcs}Y_{cs}=\frac{s_{Xdt}^{\iota-1}}{1+\kappa s_{Xdt}^{\iota}}\sum_{s=t}^{\infty}B_{s,t}\widehat{p}_{Xds}Y_{ds}, \tag{25}$$

where $\widehat{p}_{Xcs}$ and $\widehat{p}_{Xds}$ denote the shadow price of the clean and dirty inputs in country X, $\widehat{p}_{Hs}$ the shadow price of good H and $B_{s,t}=\frac{1}{(1+\rho)^{s-t}}\frac{\frac{\partial u}{\partial C}(C_{Ws},S_s)}{\frac{\partial u}{\partial C}(C_{Wt},S_t)}$, where $u\left(C_{Wt},S_t\right)=\frac{(\nu(S_t)C_{Wt})^{1-\eta}}{1-\eta}$ and $C_{Wt}\equiv C_{Nt}+C_{St}$ is world consumption. $B_{s,t}$ is the effective discount factor between period s and t. Third, the underutilization of intermediates due to monopoly pricing is corrected by a subsidy $1-\gamma$ to all intermediates. Finally, when the social planner cares about the distribution of consumption, transfers are used to equalize the social marginal value of consumption in each country (that is $\Psi C_{Nt}^{-\eta}=(1-\Psi)\,C_{St}^{-\eta}$).

Since the utility flow is minimal during a disaster and the social planner can always reduce world emissions, the optimal policy always avoid a disaster. The following remarks further characterizes the optimal policy and establishes conditions under which a switch to clean innovation occurs in one country - as in remark 1.[27]

Remark 3 The social planner always avoid a disaster. If the discount rate ρ is sufficiently small and the inverse elasticity of intertemporal substitution $\eta\leq 1$, innovation in sector G switches to mostly clean innovation and both countries reach full specialization in finite time.

Proof. See Appendix B.7 ■

Since under a disaster consumption is worthless, the social planner would rather produce no dirty input than reaching a disaster, therefore he will always ensure that it does not happen. With the inverse elasticity of intertemporal substitution $\eta\leq 1$ and a sufficiently small discount rate, the optimal policy maximizes the long-run growth rate. A switch towards clean innovation allows the polluting sector to keep growing at a positive rate while avoiding a disaster. Moreover, long-run growth is maximized if each country innovates only in its own sector (as innovations in one country and the other do not add up). In this case, the difference in comparative advantage becomes so large that both countries end-up fully specializing. Since

[27] This remark would still hold if $S=0$ was not an absorbing state.

the dirty input becomes a negligible part of the production process, emissions vanish, and the quality of the environment goes back asymptotically to $\overline{S}$. The carbon tax becomes irrelevant and can be set at 0 with very little welfare consequences.[28]

Interestingly, when the social welfare function is given by (1) - the no-redistribution concerns case - the country exporting the polluting good is not necessarily the one where consumption is reduced the most due to environmental policy. Indeed, the reduction in the production of the polluting good creates a terms of trade effect beneficial to the polluting country. When final consumption is Cobb-Douglas ($\sigma = 1$), and the policy intervention does not affect the pattern of specialization, long-run consumption is reduced proportionally in both countries relative to laissez-faire. Once countries are fully specialized, the country exporting good G has a fixed income share of ν. When the goods are strict complements ($\sigma < 1$), the country exporting good G in the long run actually ends up having a larger share of world consumption as the terms of trade effect is stronger.

4.2 Second best

I now turn to the case where the social planner cannot implement any policy in the South (the economy there must behave as in laissez-faire),[29] and cannot transfer income from one country to another (so that trade balance must be maintained at every point in time: there is no international lending). The second best policy is defined by the social planner maximizing (1) or (2) under the following constraints: (3) for the North and the South, the constraints ((4), (5), (7), (8), (9), (11), (15), (13)) for the North only; the environmental degradation constraint (16), market goods clearing in both countries, which are now written as:

$$C_{NYt} = Y_{NYt} + M_{Yt} \text{ and } C_{SYt} = Y_{SYt} - M_{Yt}, \text{ for } Y \in \{G, H\}, \tag{26}$$

where M_{Yt} denotes net imports of the North of good Y, the trade balance constraint

$$p_t M_{Gt} + M_{Ht} = 0, \tag{27}$$

where $p_t \equiv p_{Gt}/p_{Ht}$ is the international price ratio, and constraints describing the laissez-faire economy in the South. These constraints (detailed in Appendix B.8) are given by a consumer

[28] In the sense that for any $\delta > 0$, there is a T such that for $t > T$ the difference in absolute value between the utility flow with the optimal policy and a utility flow with the same policy but a carbon tax set at 0 is smaller than δ. If one had assume that $\nu(S)$ was constant on some interval around $\overline{S}$, the optimal carbon tax would be 0 in finite time.

[29] To avoid a scale effect when comparing to the first best in the calibration, I consider that the optimal subsidy for the use of all intermediates is implemented in the South. This has no bearing on any of the theoretical results since this subsidy only has a scale effect. Moreover, the equilibrium in the South must be unique given the allocation chosen by the North, therefore I assume all along that κ is sufficiently small and $\iota \geq 1/2$.

demand equation:

$$\frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial C_S}{\partial C_{SH}}} = \frac{\nu C_{SHt}^{\frac{1}{\sigma}}}{(1-\nu) C_{SGt}^{\frac{1}{\sigma}}} = p_t, \tag{28}$$

offer equations in the South of the type:

$$Y_{SGt} = y_{SG}(p_t, A_{SGt}, A_{SHt}) \text{ and } Y_{SHt} = y_{SH}(p_t, A_{SGt}, A_{SHt}), \tag{29}$$

an emissions equation $Y_{Sdt} = (A_{Sdt}/A_{SGt})^{\varepsilon} Y_{SGt}$, an equation giving the total mass of scientists allocated to sector G,

$$s_{SGt} = s_{SG}(p_t, A_{Sdt}, A_{Sct}, A_{SHt}), \tag{30}$$

and the resulting law of motion of aggregate productivity in the South - where the allocation between clean and dirty innovation is uniquely determined given the total mass s_{SGt} and the ratio $A_{Sc(t-1)}/A_{Sd(t-1)}$-:

$$A_{SHt} = \left(1+\kappa\left(1-s_{SGt}\right)^{\iota}\right)^{1-\gamma} A_{SH(t-1)}, \tag{31}$$

$$A_{Szt} = \left(1+\kappa\left(s_{Szt}\left(s_{SGt}, \left(\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}\right)^{\varepsilon-1}\right)\right)^{\iota}\right)^{1-\gamma} A_{Sz(t-1)}, \text{ for } z \in \{c,d\} \tag{32}$$

The second best policy can be decentralized in the following way:[30]

Proposition 4 *The second best policy can be decentralized through a carbon tax in the North, research subsidies/taxes in the North, a subsidy to the use of all intermediates and a trade tax.*

Proof. See Appendix B.8 and D.5 ■

In this second best scenario, the social planner uses the same instruments as before to address the inefficiencies of the economy in the North: a subsidy to the use of all intermediates (for the monopoly distortion), a carbon tax, and research subsidies in order to allocate scientists as in (25). The trade tax is the optimal way to affect prices in the South, which is the only channel through which the social planner can intervene on the South economy. In Appendix B.8, I derive an implicit equation for the value of the optimal trade tax (95) for the maximization of (1), which takes the following form:

$$\begin{aligned} & f_t(b_t) \\ = \; & c_{1t}\tau_{Nt} + c_{2t}\left(\frac{s_{SHt}^{\iota-1}\mu_{SHt+1}A_{SHt+1}}{1+\kappa s_{SHt}^{\iota}} - \frac{\partial s_{Sdt}}{\partial s_{SGt}}\frac{s_{Sdt}^{\iota-1}\mu_{Sdt+1}A_{Sdt+1}}{1+\kappa s_{Sdt}^{\iota}} - \frac{\partial s_{Sct}}{\partial s_{SGt}}\frac{s_{Sct}^{\iota-1}\mu_{Sct+1}A_{Sct+1}}{1+\kappa s_{Sct}^{\iota}}\right) + term_{3t} \end{aligned} \tag{33}$$

[30] For $\sigma < 1/2$, there may be several equilibria corresponding to a given policy, the social planner would need to be able to directly choose the amount of imports - through quotas for instance - in order to pick the right equilibrium.

where $f_t(b_t)$ is an expression that has the sign of b_t, c_{1t} and c_{2t} are positive coefficients when the South is not specialized and null otherwise (when the South is at a corner of specialization, the coefficients can be positive or null), and c_{3t} and $\mu_{Sz(t+1)}$ denotes the social value of a productivity unit in the South in (sub)sector $z \in \{c, d, H\}$. The first term is always positive and represents the environmental motive for the trade tax: pollution in the South escapes direct taxation, and a positive trade tax on the polluting good G (a tariff or an export subsidy) reduces the relative price of good G in the South, which reduces its production and therefore emissions in the South. The second term is a correction for the myopia of innovators in the South; it represents the difference between the social value of an additional scientist in the non-polluting sector H and one in the polluting sector G on welfare for all subsequent periods. In principle, it could be of either sign, but in practice it ends up being positive, also pushing towards a positive tariff or export subsidy. Indeed, to avoid a disaster, the South must at least asymptotically fully specialize in the non-polluting sector (see lemma 3). Therefore, whenever the optimal policy avoids a disaster, current innovations in the polluting sector are of little use in the future. A positive trade tax on the polluting good G tilts innovation in the South away from that sector. The third term ($term_{3t}$) has an ambiguous sign but goes to zero for a sufficiently large gap between clean and dirty technologies.[31] Overall, the trade tax is generally positive in this case, taking the form of a tariff when the North imports the polluting good and of an export subsidy when it exports it.

In the case of the maximization of (2) there is a forth term in the right-hand side of (33): $c_{4t}\left(1-\frac{\lambda_{St}}{\lambda_{Nt}}\right)\left(\frac{C_{SHt}}{C_{SGt}}\frac{Y_{SGt}}{Y_{SHt}}-1\right)$, where c_{4t} is positive and λ_{Xt} is the Lagrange multiplier associated with (3) in country X and represents the marginal social value of a unit of consumption at time t in country X. This last term represents the terms of trade motive, the trade tax is modified in order to favor the sector with the largest social marginal value of consumption. When the social planner cares only about the North ($\Psi = 1$), this would push towards a higher tariff when the North imports the polluting good and an export tax otherwise. When the social planner cares equally about both countries ($\Psi = 1/2$), but the South is poorer, it would push towards first an import or an export subsidy.[32]

To better relate my analysis with the literature, I derive in Appendix D.4 the following condition for the social optimum:

$$\frac{p_t + M_{Gt}\frac{\partial p_t}{\partial M_G}}{1 + M_{Gt}\frac{\partial p_t}{\partial M_H}} = \frac{\frac{\partial C_{Nt}}{\partial C_{NGt}} + \left(\frac{\phi_t}{\lambda_{Nt}}\frac{\partial s_{SGt}}{\partial p_t} - \frac{\omega_t}{\lambda_{Nt}}\frac{\partial Y_{SGt}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_{Gt}} - \frac{\lambda_{St}}{\lambda_{Nt}}\frac{\partial C_{St}}{\partial C_{SGt}}}{\frac{\partial C_{Nt}}{\partial C_{NHt}} + \left(\frac{\phi_t}{\lambda_{Nt}}\frac{\partial s_{SGt}}{\partial p_t} - \frac{\omega_t}{\lambda_{Nt}}\frac{\partial Y_{SGt}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_{Ht}} - \frac{\lambda_{St}}{\lambda_{Nt}}\frac{\partial C_{St}}{\partial C_{SHt}}} \tag{34}$$

[31] This term is fairly technical. It reflects that if more Southern scientists are allocated to sector G today, then for a given number of scientists allocated to sector G tomorrow, more will be allocated to dirty than to clean technologies.

[32] In that case the sign of the second term could be ambiguous as the social value of future innovations includes how innovation in the South will affect the future terms of trade between the two countries.

where ϕ_t is the social value of moving an infinitesimal mass of scientists in the South to sector G. at time t and $\lambda_{Nt} = \lambda_{St} = \lambda_t$ in the case of maximizing (1). $M_{Gt}\frac{\partial p_t}{\partial M_G}$ and $M_{Gt}\frac{\partial p_t}{\partial M_H}$ measure the terms of trade effect of an additional unit of imports in sector G and H respectively. This equality stipulates that the ratio of the cost of imports for the North (prices plus terms of trade effects) must be equal to the ratio of marginal social benefit, which include more consumption of the imported good in the North, less consumption in the South weighted by its relative marginal social value, environmental damage, and the impact on innovation.

The next proposition further characterizes the shape of the optimal policy. It derives conditions under which the social planner tries to avoid a disaster and then conditions under which a switch to clean innovation occurs in the North, so that the optimal policy looks similar to the policy described in subsection 3.4.

Proposition 5 *(i) The social planner avoids a disaster whenever it is feasible if the inverse elasticity of intertemporal substitution $\eta \geq 1$, or if $\eta < 1$ and the discount rate ρ is sufficiently small. The South must asymptotically be fully specialized in the non-polluting sector H if initially clean technologies are less developed than dirty ones there $A_{Sc0} \leq A_{Sd0}$.*
(ii) Further if $A_{Sc0} \leq A_{Sd0}$, avoiding a disaster is feasible, ρ is sufficiently small, and either the inverse elasticity of intertemporal substitution $\eta \leq 1$, or the polluting and non-polluting goods are strict complements ($\sigma < 1$), there is a switch towards clean innovation in the North: the mass of scientists allocated to the dirty subsector tends to 0 and the mass of scientists allocated to clean technologies in the North is positive (asymptotically 1 if $\eta \leq 1$).

Proof. See Appendices B.9 and D.6 ■

The intuition behind this proposition is similar to the intuition behind remark 3, although avoiding a disaster is not always feasible if the initial environmental quality is too low. When the inverse elasticity of intertemporal substitution $\eta \geq 1$, a disaster brings a utility of $-\infty$ so the social planner always tries to prevent it. He also does so for a sufficiently small discount rate ρ and $\eta \leq 1$ as in this case a social planner tries to maximize long-run growth which is impossible under a disaster.[33] To avoid a disaster, the South must asymptotically specialize in the non-polluting sector H if it relies on dirty technologies in the polluting sector. Maximizing long-run growth requires that the North innovates only in clean technologies asymptotically - as the South innovates in sector H only asymptotically. When $\eta > 1$ and the discount rate is sufficiently small, one can show that the social planner always prefer a path with positive long-run growth to one with bounded consumption. Positive long-run growth can be achieved only with growth in both sectors when the two goods are strict complement ($\sigma < 1$), so in this

[33] If the disaster was not an absorbing state, a temporary disaster could be part of the optimal policy for $\eta < 1$.

case too as switch to clean innovation must occur in the North. In both cases, the optimal policy looks like the one described in subsection 3.4 (the alternative to avoid a disaster would be to not develop clean technologies in the North and to keep production of the polluting good bounded with the carbon tax). This is particularly true for the case of maximizing (1): in this case, under the assumptions in (ii) in proposition 5, both countries fully specialize in finite time, innovation in the North is asymptotically all in clean technologies, the trade tax is temporary and as the environment recovers, the carbon tax becomes irrelevant.[34]

Alternatively, one could look at a case where income transfers are allowed across countries, so that the constraint (27) is removed from the problem. This case is similar to the one just studied but the trade tax is not affected by redistributive concerns.

5 Stylized Calibration

In this section I carry a simple calibration exercise. This exercise should not be taken as a careful quantitative assessment, as the level of aggregation of the model is too high for this purpose, but as an illustration of the theoretical results above. In particular, I show how both trade and directed technical change act as "double-edge swords": they tend to accelerate environmental degradation in laissez-faire and when the North undertakes only non-protectionist policies, but they help prevent a disaster when the North undertakes the appropriate policies.

5.1 Parameter choices

I provide a brief description of the calibration, further details are given in Appendix C. I define 1 period as 5 years and the initial values are based on the world economy in 2003-2007, assuming laissez-faire in both countries. For simplicity, I assume that the polluting and non-polluting goods enter in final consumption in a Cobb-Douglas way ($\sigma = 1$), and that the elasticity of intertemporal substitution is equal to 1 ($\eta = 1$) - as in Golosov et al. (2011). The annual time discount rate is 0.015 as in Nordhaus (2008). I identify the North with the countries in Annex I of the Kyoto protocol (the countries submitted to binding constraints on their emissions) and the South as the rest of the world. Data availability restricts the number of countries studied but the sample includes the most important countries (the North comprises 33 countries and the South 18). To identify the polluting and non-polluting sectors, I rely on the IEA data on sectoral emissions of CO2 from fossil fuel combustion across the world (IEA (2010*a*)) and on the UNIDO data on sectoral value added (UNIDO (2011)). I restrict attention to manufacturing and compute at the available aggregation level the world rate of emissions per dollar of value added in each sector. The sectors with the highest rate are identified

[34]For the maximization of (2) when the North has all the weight ($\Psi = 1$), the South must asymptotically be at the corner of specialization, and the trade tax tends to -1. See Appendices B.9 and D.6 for proofs.

with sector G and the other ones with sector H (and, according to the model, I ignore the emissions coming from sector H). In practice, the polluting sector corresponds to manufacture of chemicals and chemical products (ISIC code 24), of other non-metallic mineral products (26) and of basic metals (27).[35] I find that the South production is tilted towards sector G relative to the North production ($Y_{NG0}/Y_{SG0} \times Y_{SH0}/Y_{NH0} = 0.77$), which in the framework of the model corresponds to the South having a small initial comparative advantage in the polluting sector G. From world production in sectors G and H, I compute the consumption share of good G ($\nu = 0.257$) when the economy consists only of sectors G and H.

I compute the capital factor share from the ratio of capital compensation over labor compensation in both sectors in the US with the EU KLEMS dataset (Timmer, O'Mahony and van Ark (2008)), and find a capital share $\alpha = .5$ in the polluting sector, and $\beta = .3$ in the non-polluting one. A unit of a good is defined as the quantity with a value added worth one billion of dollars in 2000. Factor shares and initial production values are enough to determine the initial productivity adjusted endowments, which, with the initial ratio A_{Xc0}/A_{Xd0}, are all that matter for the economy when knowledge is purely local.[36] I fix $\gamma = 1/3$, which is a common value in endogenous growth models. For the elasticity of substitution between the clean and the dirty input ε, I choose $\varepsilon = 5$, the medium value used in the working paper version of AABH.[37] For the innovation function I choose $\iota = 1/4$ (numerically this parameter has a limited impact) and κ is adjusted so that the long-run growth rate of the economy is 2% a year.

The quality of the environment S_t is linearly negatively related to the atmospheric concentration of CO_2 (the assumption that $S_0 = \overline{S}$ is relaxed and the initial environmental quality S_0 is set such that it corresponds to the current atmospheric concentration of 379 ppm). Δ is calibrated such that at current levels around half of CO_2 emissions are absorbed and do not add to atmospheric concentrations. I compute the emission rate in the North and in the South in sector G, the South's rate is nearly four times as large as the North's. Such a large difference cannot be accounted for by the model if $A_{Nd0} > A_{Nc0}$ and the emission rate per unit of dirty input (ξ) is identical in both countries. Since $A_{Nc0} > A_{Nd0}$ would be very unrealistic, I relax the assumption that the emission rates per unit of dirty input are the same in both countries. To derive a proxy for A_{Xc0}/A_{Xd0}, I use IEA data (IEA (2010*b*)) and identify the ratio Y_{Xc0}/Y_{Xd0} with the ratio of the production of nonfossil fuel energy over fossil fuel energy

[35] Sector H corresponds to the other sectors in manufacturing except 23, 25, 33, 36 and 37 for which data are not available.

[36] Nevertheless, I do not assign arbitrary values for endowments, but choose L_X as total employment in sector G and H in country X, and K_X as total capital formation in both sectors in country X (from the UNIDO database).

[37] There is no good empirical estimation of this parameter yet. Its important role is the focus of the numerical exercise in AABH, it does not strongly affect the comparative statics exercise carried out here.

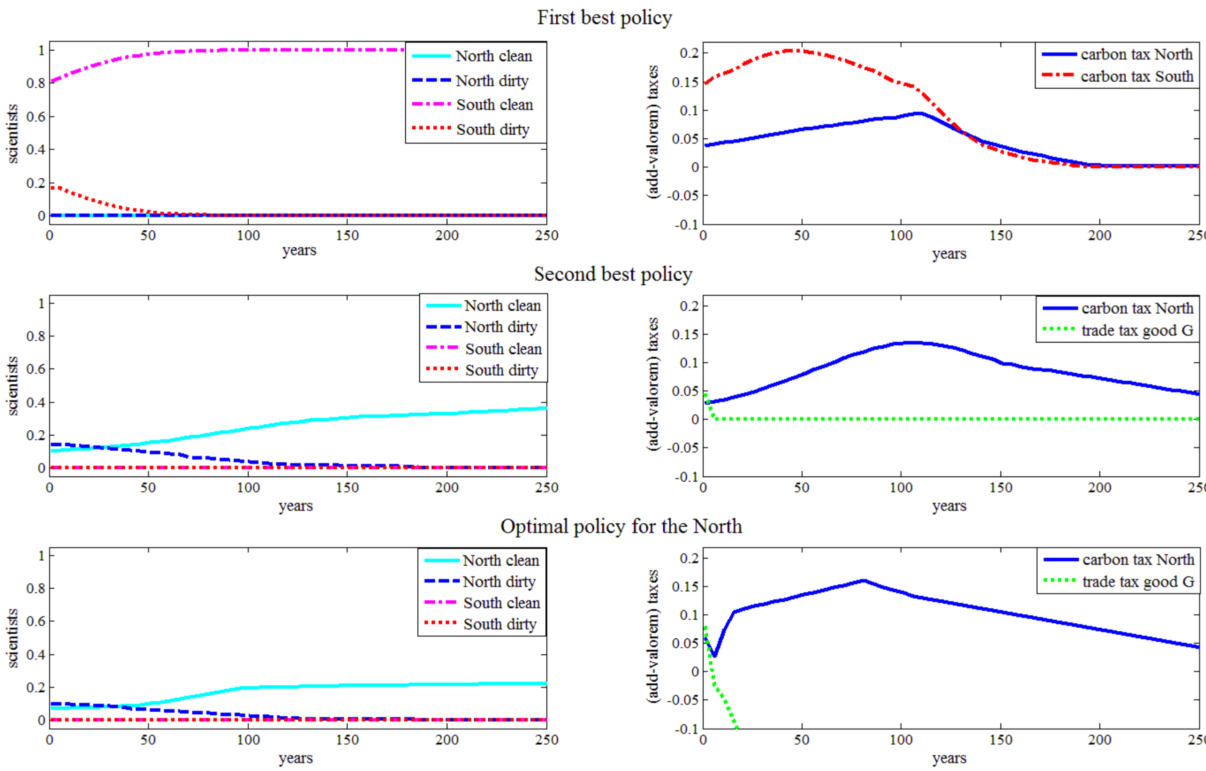

Figure 1: Figure 1: First best, second best and North optimal policies

in country X's primary energy supply. This ratio is 25% larger for the North than for the South. The rest of the difference between the initial emission rates per unit of good G in the North and in the South is made up by the difference between ξ_N and ξ_S ($\xi_S > \xi_N$ so dirty inputs in the South are more polluting than in the North).[38] Changes in CO_2 atmospheric concentrations are then mapped with changes in temperature, and $S = 0$ is chosen to correspond with a "disaster" temperature level of 6°C. The function $\nu(S_t)$ is the same as in AABH, and mimics the cost function of Nordhaus (2008) for increases up to 3°C.

5.2 Alternative policies and their welfare costs

Figure 1 displays the allocation of innovation, carbon tax and trade tax in the first best case, the second best case for (1) (referred to as "Second best policy" in the graph), and the second best case for (2) when the weight of the North is 1 (referred to as "Optimum policy for the North"). Figure 1.A shows the allocation of innovation in the polluting sector G in the first best

[38] Finally, ξ_N and ξ_S are adjusted upwards so that emisssions in sector G in the North and in the South correspond to total world emissions.

case: nearly all sector G innovation is carried out in the South, and mostly in clean technologies from the first period. Following remark 3; specialization in both countries occurs in finite time, in fact here, it occurs very rapidly, which is why there is nearly no innovation in sector G in the North (and nearly no innovation in sector H in the South). Figure 1.B shows the ad valorem carbon taxes in the North and the South - recall that the per-unit of CO_2 taxes are identical though. The carbon tax in the South initially increases (as temperature increases) and then decreases following the fall in temperature and the development of clean technologies in the South. Figure 1.C shows the allocation of innovation in the second best case. Contrary to the first best case, the North must now export the polluting good G eventually. In this calibration, a large trade tax on good G (see figure 1.D) ensures that right from the first period the South specializes in the non-polluting good H.[39] The switch towards more clean innovation than dirty in the North occurs rapidly but not immediately for two reasons: first, as the South's emission rate is larger than the North's, the temperature increase is initially lower than in the first best case, so the North can afford working on dirty technologies a bit longer; second, keeping some innovation in dirty technologies helps the North build a large comparative advantage in the polluting sector. The amount of clean innovation increases over time and, beyond the time frame of the simulation, eventually reaches one when the North fully specializes in the polluting sector (following proposition 5). The carbon tax in figure 1.D follows a pattern similar to the first best case, however, since clean technologies are developed slower, emissions decrease less fast (see figure 2.A) and the carbon tax goes to zero slower. Finally, figure 1.E shows the allocation of innovation in the North optimal policy. The pattern is similar to the second best case, but less North scientists are allocated to the polluting sector. Since the North becomes the exporter of good G, allocating less scientists to its export sector positively affects its terms of trade. In this calibration, the South is also fully specialized from the first period, but must remain just at the corner of specialization. To maintain this situation the trade tax turns negative from the second period, and becomes an export tax on the polluting good for the North (for readability the figure is cut, but as the comparative advantage increases, the trade tax tends towards -1, that is a 100% tax on exports). To cope with the smaller innovation in the polluting sector in the first period in the North, the trade tax is larger in the first period. The fast specialization in at least one country that occurs in this calibration can be explained by a relatively large growth rate (2% a year) combined with a small difference in capital share in both sector ($\alpha - \beta = .2$) and a small initial comparative advantage. Taking into account imperfect mobility of factors or both cross-sector and cross-country knowledge spillovers would slow down the specialization process.

[39] In this case the initial trade tax is sufficiently large to reverse the pattern of trade (see footnote 16).

Table 1: disaster and welfare cost

	time until disaster	welfare cost
Miracle	never	0%
Laissez-faire	80 years	100%
First best	never	5.64%
Second best	never	14.08%
Third best	never	14.22%
Non protect.	80 years	100%

Table 1 shows whether a disaster can be avoided under different scenarios and what the welfare costs of climate change are. The welfare cost is computed as the equivalent percentage loss of world consumption every period relative to the first best in a “miracle” scenario where the dirty input would cease to pollute (that is from the first period $\xi_N = \xi_S = 0$). Under laissez-faire a disaster occurs after 80 years (the welfare cost is 100% with log utility). There is no disaster in the first best case (which is always true) and since initial environmental quality is sufficiently large, there is no disaster either in the second best case. Not being able to intervene in the South, and therefore having to reverse the pattern of trade, sharply increases the welfare costs of climate change policy (they are nearly three times as large).[40] Table 1 also presents the case of a “third” best where the North can implement a carbon tax and research subsidies/taxes, but no trade tax, consumption or production taxes. With the calibrated parameter values, a tariff happens to be not necessary to avoid a disaster and the welfare costs of dispensing with it are relatively small: This is because the initial comparative advantage of the South in the polluting sector is small. No combination of a carbon tax and a tax on dirty research in the North can prevent an environmental disaster, and in table 1, I compute the combination of a carbon tax and a tax on dirty research (“non-protectionist policy”) that minimizes the amount of CO_2 accumulated. This policy cannot postpone the disaster, and as shown in figure 2 below, its effect on temperature is extremely small relative to laissez-faire.

5.3 Trade, a double-edged sword

Figure 2 shows the increases in temperature in laissez-faire, with the non-protectionist policy that minimizes the amount of CO_2 emissions, in the second best case and in the first best case, and then compares them with the increases in temperature with the same policies in autarky. In line with table 1, non-protectionist policies cannot slow down the disaster; in fact, the difference in temperature is so small that the two curves are indistinguishable. In the first best case the increase in temperature is initially larger than in the second best case, the reason is

[40] Note that this increase in cost is almost entirely due to the environmental externality. In the miracle case, there would also be some welfare costs from not being able to intervene in the South - as innovation there would not be allocated optimally- but these costs would be very small: 0.15%.

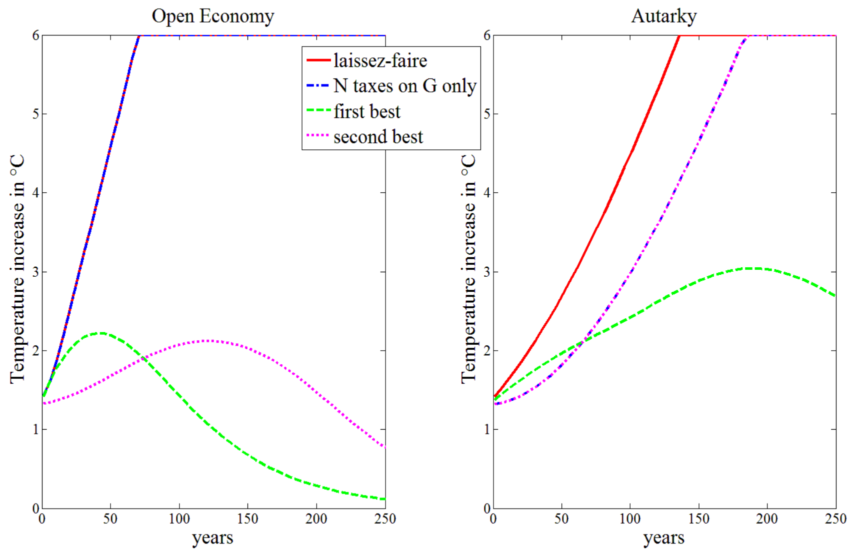

Figure 2: Figure 2: Increases in temperature with trade and in autarky

that in the latter case production of the polluting good moves to the North, where the emissions rate is lower. The temperature starts decreasing earlier in the first best case than in the second best case because (nearly) all Southern scientists innovate in clean technologies in the first best policy, but not all scientists in the North do so in the second best one (see figures 1.A and 1.C). In autarky, the disaster is postponed, as the polluting sector grows slower (a mass of around 0.2 scientists innovate in the polluting sector in autarky, instead of all scientists of one country in free trade). Non-protectionist policies in the North (a carbon tax and a tax on dirty research) can now postpone a disaster as the pollution haven effect does not exist, but whether clean research subsidies are allowed for or not does not really affect the increase in temperature. In figure 2.B, in the "second best" case (which now refers to the combination of carbon taxes and clean research subsidies in the North that minimizes CO_2 emissions), the temperature increase cannot be distinguished from the temperature increase resulting from taxes on the polluting sector in the North only. Even in the first best case temperature increases more as the growth rate of clean technologies is smaller than in the open economy scenario. Overall, figure 2 illustrates the double-edged sword role of trade: without trade, unilateral policies cannot prevent a disaster, but opening up to trade accelerates environmental degradation if the North does not undertake the appropriate policy.

5.4 Directed technical change, a double-edged sword

Directed technical change (DTC) plays a similar role. With the calibrated values, however, non-protectionist policies cannot delay a disaster either when there is no DTC and in both

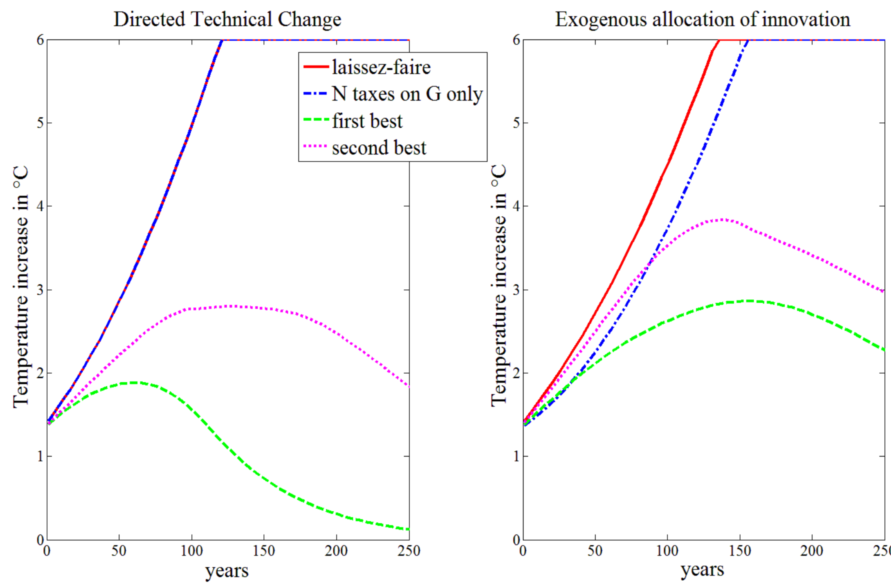

Figure 3: Figure 3: Increases in Temperature with and without Directed Technical Change

countries $s_c = s_d = s_H = 1/3$. To better illustrate the impact of DTC, I carry the same exercise but assuming that $\alpha = 0.7$ and $\beta = 0.1$ (a larger difference in capital limits the pollution haven effect in a static model and therefore better illustrates how it is amplified by the innovation response). The resulting increase in temperature across the different policies and under the two scenarios are given in figure 3 (Appendix C.2 carries the exercise with the original values). DTC accelerates the disaster under laissez-faire since it accelerates the growth rate of the production sector. Without DTC, non-protectionist policies are still unable to prevent a disaster, but they can delay it for 20 years. With a permanent and large trade tax, unilateral policies can still avert an environmental disaster with these parameters - recall that this is not generally true - but the increase in temperature is much larger (despite a much lower growth rate), and even in the first best case temperature increases for a longer time.

5.5 Distributional impacts

Figure 4 shows the trade tax on the polluting good and South consumption under the following regime: laissez-faire, the second best for (1) -"no distribution concern"- and the second best for (2) when the North has all the weight -"North optimal"- and when the countries have equal weight -"Equal weight-. Note first that in the no redistribution concern case the trade tax remains small relative to the other cases where the North has all the weight or where the weight is equally shared (since the South is poorer to start with, terms of trade are distorted in favor of the South in that case). Second, at equal weight, the social planner would want

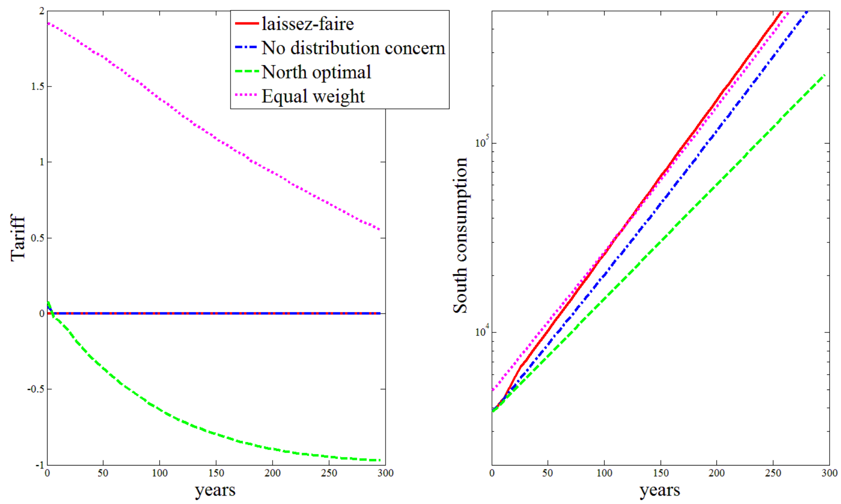

Figure 4: Figure 4: Distributional impacts

to implement a trade tax sufficiently large to reverse the pattern of trade, so that the North exports the polluting good, and the trade tax is an export subsidy. This very large trade tax benefits the South: its consumption flow is in fact larger than in laissez-faire until year 250. Third, and perhaps most surprising, in the first period, the trade tax in the "no distribution case" is also suffiicenty high to be revert the pattern of trade and actually benefits South's consumption (it is higher than the laissez-faire case in the first period).

6 Knowledge diffusion

I now relax the assumption that productivity improvements are entirely country specific. The amplification of comparative advantage effect has been one of the dragging forces behind the previous results. In reality, some productivity improvements are likely to at least partly cross borders, so that some economic forces will work against this effect.[41] This raises the issue of the robustness of the previous analysis. Here I consider an extension of the original model, where the laggard country can benefit from the diffusion of innovations produced in the other country. In Appendix A.2, I present another extension of the model where innovation itself is international and done by global firms. In both cases the main messages of section 3 still hold, even though the underlying intuitions are somewhat different.

To illustrate knowledge diffusion in a simple way, I assume that at the beginning of every

[41] However, one should not expect all productivity improvement to easily cross borders, as they may be embedded in capital and infrastructures, or may depend on local know-how. Dechezleprêtre et al. (2010) suggest that clean technologies transfer between developing and developed countries exist but are quite limited.

period the country with the least advanced average productivity in a given sector can exogenously partially catch up with the other country. More specifically, before any innovation happens, the producer of intermediate i in sector $z \in \{c, d, H\}$ gets access to the technology:

$$\overline{A_{Xzit}} = \max\left(\left(\frac{A_{(-X)z(t-1)}}{A_{Xz(t-1)}}\right)^{\delta}, 1\right) A_{Xzi(t-1)},$$

where $\delta \in [0, 1]$ measures the strength of the technological diffusion. This formulation delivers in return the law of motion for aggregate productivity

$$A_{Xzt} = (1 + \kappa s^{\iota}_{Xzt})^{1-\gamma} \max\left(\left(\frac{A_{(-X)z(t-1)}}{A_{Xz(t-1)}}\right)^{\delta}, 1\right) A_{Xz(t-1)},$$

for $z \in \{c, d, H\}$. With this formulation, the ratio of the technological levels across countries cannot diverge: when one country gets a sufficiently strong advantage over the other one, the catching up process ensures that regardless of the pattern of innovation, this difference is reduced next period.

In particular, policies in the North which increase the amount of clean innovation in the North will now also increase the productivity of clean technologies in the South. In fact, they may even put the South on a clean innovation track: if at one point in time pre-innovation clean technologies are above dirty ones in the South $\overline{A}_{Sct} > \overline{A}_{Sdt}$, market forces in the South will induce more clean than dirty innovations. The key to preventing a disaster is no longer to push the South towards specializing in the non-polluting sector, but rather to ensure a switch towards clean innovation in the South. Such transition will occur as soon as more scientists are allocated to clean technologies in the North than to dirty technologies in the South for a sufficient amount of time (since in the long-run, South clean productivity A_{Sct} grows like the North one A_{Nct}). Whether this is the case or not directly depends on the policies that the North allows for and on the pattern of comparative advantage, similarly to the analysis in section 3. Therefore, the intuitions developed before still apply and, surprisingly, the broad results are not as different as one could have expected. In particular, I can show:

Proposition 6 *Assume that initially (i) technologies are sufficiently close to each other across countries, that κ is sufficiently small and the spillovers δ are sufficiently strong, (ii) that the South is relatively well endowed in capital $\frac{K_S}{L_S} > \frac{K_N}{L_N}$, and that (iii) clean technologies are sufficiently less advanced than dirty ones (A_{Sc0}/A_{Sd0} sufficiently small), then no combination of a carbon tax and a tax on dirty research in the North can prevent a disaster, no matter how large $\overline{S}$ is.*

Proof. See Appendix D.7 ■

This proposition mirrors proposition 1. The assumptions (i) ensure that technological levels remain sufficiently close to each other across countries, which combined with the assumption (ii) ensures that the South keeps a comparative advantage in the polluting sector - assumption (iii) plays the same role as in proposition 1 and ensures that when the South has the comparative advantage in the polluting sector, it does innovate there more than the North; this assumption can be dispensed with in the perfect substitution extension (with $\varepsilon = \infty$). In this case, the South keeps the comparative advantage in the polluting sector, and since a carbon tax in the North can only reinforce this comparative advantage; more scientists are innovating in dirty technologies in the South than in clean in the North: South clean productivity $\overline{A}_{Sct}$ never catches up, and a switch to clean innovation in the South never occurs. In other words, the market for the polluting good in the North is too small to generate enough clean innovations.

As before, a temporary combination of clean research subsidies and a tariff can prevent a disaster for sufficiently large initial environmental quality (proposition 2 still holds): clean research subsidies can reallocate innovation in the North to clean technologies while a tariff can limit innovation in dirty technologies in the South, so that $\overline{A}_{Sct}$ grows faster than $\overline{A}_{Sdt}$ and a switch towards clean innovation eventually occurs in the South.

Remark 2 is not robust when clean and the dirty inputs are not perfect substitutes. Sufficiently large clean research subsidies in the North alone can now avoid a disaster if the initial environmental quality is sufficiently large even when final consumption is Cobb-Douglas in the polluting and non-polluting goods ($\sigma = 1$).[42] Since clean technologies in the South grow at the same rate as in the North, the ratio of clean over dirty technologies in the South cannot tend towards zero if the North allocates all its scientists to clean technologies. As a result, the mass of scientists allocated to dirty technologies in the South remain bounded away from one when the South specializes in the polluting sector. Eventually, pre-innovation clean productivity in the South becomes larger than dirty productivity, $\overline{A}_{Sct} > \overline{A}_{Sdt}$, and a switch towards clean innovation must occur.

This analysis relies on the innovation function κs^{ι} satisfying the Inada condition. If instead, the innovation function were $\kappa\left((s + \Upsilon)^{\iota} - \Upsilon^{\iota}\right)$ with $\Upsilon > 0$, then, for clean technologies initially sufficiently less advanced than dirty ones (A_{Sc0}/A_{Sd0} being sufficiently small), all innovation in the South could go towards dirty technologies when the South is fully specialized in the polluting sector. The same scenario applies to the perfect substitute alternative ($\varepsilon = \infty$). In this situation, whether clean research subsidies alone can prevent a disaster or not in the Cobb-Douglas case ($\sigma = 1$) depends on how large the difference in relative factor endowment is. For sufficiently close initial endowments, knowledge spillovers guarantee that the South does not specialize in the long-run, which prevents all scientists from being allocated to dirty

[42] For $\sigma < 1$, clean research subsidies can avoid a disaster alone for sufficiently large initial environmental quality as in the no spillovers case.

technologies in the South. As a result clean technologies can catch-up in the South and a disaster can be avoided. On the contrary, if initial endowments are sufficiently far apart, perfect specialization of the South in the non-polluting sector can be maintained indefinitely. Therefore, the South only innovates in dirty technologies and a disaster cannot be avoided without a trade tax. A formal proposition is given and proved in Appendix D.7.[43]

The structure of the optimal policy (with or without the no intervention in the South constraint) is broadly similar, but subsidies to research and the trade tax will have to take into account the presence of the knowledge spillovers. Moreover, when the second best policy prevents a disaster, it does not necessarily feature a South exporting the non-polluting good in the long-run.

Note that to some extent technological diffusion itself is a parameter that can be affected by policy: laxer intellectual property rights, direct financing of projects abroad, or migrations of skilled workers could all contribute to the faster diffusion of technology. This analysis therefore suggests that the diffusion of clean technologies in the South makes the need for a tariff less pressing. In fact, since the high social welfare cost from unilateral intervention without knowledge spillovers came from the necessary reversal of comparative advantage, diffusing technologies in order to prevent a disaster without such a reversal in comparative advantage could reduce significantly the costs of the intervention.

7 Conclusion

This paper develops a dynamic model of trade and the environment with directed technical change in a two country world, in order to study what type of unilateral policies could achieve sustainable growth. It also characterizes the second best policy when intervention is impossible in one country. When knowledge is local, a combination of temporary clean research subsidies and a carbon tariff can prevent an environmental disaster, while unilateral taxes on the polluting sector are unlikely to do so, particularly when the South has initially the comparative advantage in the polluting sector. The second best policy can be decentralized through research subsidies, a carbon tax and a trade tax. When the social planner does not care about the distribution of consumption, the trade tax typically takes the form of a tariff on the polluting good and then of an export subsidy, reflecting the double objective of reducing emissions in the South and of redirecting innovation there. Otherwise, the trade tax is also used to affect terms of trade. Under some assumptions on the preferences of the social planner, the second best policy features a switch to clean innovation in the North, and the South must (asymptotically) specialize in the non-polluting sector. In the presence of knowledge spillovers, or with

[43] This reasoning would also apply to the default assumption $\varepsilon < \infty$ and $\Upsilon = 0$ if the North could not innovate in clean technologies when it does not produce clean intermediates.

international innovating firms, a switch to clean innovation in the South can be achieved with policies in the North, so that a disaster can be avoided without the South having to specialize in the non-polluting sector. In both cases unilateral taxes on the polluting sector may still fail at preventing a disaster.

In practice, taken at face value, a full revival of industries like metallurgy in developed countries, which is what the basic model argues for, may look unrealistic. More practically, the paper argues for an industrial policy in the North that aims at cleaning the polluting sectors, without losing to much competitiveness to the South, in order to slow down the move of polluting industries there. In particular, the paper shows the shortcomings of a "carbon tax"-only policy (or equivalent policies like a cap-and-trade) in the North in the presence of imperfect knowledge markets and a non-cooperative South. However, this aggressive protectionist unilateral policy could phase out once clean technologies diffuse to the South, or once a global agreement is found. The paper also develops the argument that unless there are direct incentives to clean research, clean technologies in the polluting industries are unlikely to be developed in the North, if the production of polluting goods move to the South (for instance because of a carbon tax without a trade barrier).

This analysis is very much a first step and could be enriched in several directions. First, I have only considered a global social planner faced with the constraint of no intervention in the South. A better understanding of climate negotiations would require modelling two competing social planners in a Nash equilibrium. As mentioned in the text, the design of the trade tax (whether it is directly related to average carbon content in the country or not) will then affect the behavior of the South planner. Second, to analyze more carefully the actual implementation of carbon tariffs, one would have to take into account WTO constraints and the possibility for firms or countries to hide the true level of their emissions or to manipulate environmental policies for their own advantage. Third, I have not investigated the issue of intellectual property rights (IPR), which may play a major role in the development of clean technologies. On one hand, laxer IPR could lead to a faster diffusion of clean technologies to the South, which facilitates a switch towards a clean path there. On the other hand, they may reduce the incentive to develop clean technologies in the North in the first place. Finally, the paper suggests that directed technical change makes emissions in the South much more responsive to policies in the North in the long-run. This calls into question the common estimates of the carbon leakage rate which are obtained in static models. Therefore, integrating directed technical change into a full numerical model of the world economy would be very useful in order to reevaluate the impact of carbon taxes and carbon tariffs.

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8 Appendix A

8.1 A.1 Extensions of section 3.

Clean input production subsidy. Section 3 focused on the role of research subsidies, carbon taxes and trade taxes, since they are the instruments used to decentralize the optimal policy. In practice, other instruments could be used in a unilateral policy (especially if tariffs were forbidden), and it is interesting to know whether a disaster could be avoided with them. For instance, a temporary direct production subsidy to the clean input could avoid an environmental disaster alone (provided that the initial environmental quality is large enough). In a first phase the North could implement a subsidy sufficiently large that the North essentially produces the clean input and clean intermediates only (this is the same as an infinite tax on any other type of production), which guarantees that the North has the comparative advantage in the polluting sector. As the North innovates in clean technologies only, while the South innovates in both sectors, the North can acquire a comparative advantage in the polluting sector even without the subsidy. At which point a disaster can be avoided following the same logic as in proposition 2.[44]

Different mass of scientists in the North and the South. If the mass of scientists in the North was much smaller than in the South, the North would eventually become a small economy relative to the South, and the South's economy will behave as if it were in autarky: regardless of the policies undertaken by the North, a disaster would be unavoidable. On the contrary, if the mass of scientists in the North was much larger than in the South, a disaster could be avoided using clean research subsidies without the need for a tariff: even when the South fully specializes in sector G, A_{NGt} would grow faster than A_{SGt}, so the North could build a comparative advantage in the polluting sector. In fact, depending on parameters, a disaster may also be avoided using taxes on dirty research or a carbon tax under the assumptions of proposition 1.[45]

Limited policy in the South. I now discuss the case where intervention in the South is possible but limited by a low fiscal capacity (the South does not have the necessary infrastructure to enforce full collection of taxes beyond some threshold). More specifically, the size of government revenues in the South cannot exceed some faction g of the size of the econ-

[44] Another possibility is that the North could only implement a consumption tax on the polluting good instead of a tariff. Such a tax can prevent the South from specializing in the polluting good (by reducing its relative price), but may induce the North to (temporarily) specialize in the non-polluting sector, in which case clean intermediates are no longer produced in the North for some periods. Assuming that entrepreneurs can still hire scientists even when they do not produce any intermediates, a combination of the consumption tax and clean research subsidies can avoid a disaster for sufficiently large initial environmental quality.

[45] For instance, in the Cobb-Douglas case $\sigma = 1$, if the consumption share of the polluting good (ν) is close to 1, $\frac{A_{NGt}}{A_{NHt}}$ can grow faster than $\frac{A_{SGt}}{A_{SHt}}$, leading to a reversal of comparative advantage, provided that there is a sufficiently large mass of scientists in the North; if $\nu = 1/2$, on the contrary, the previous analysis carries on.

omy (measured by $p_{Gt}Y_{SGt} + p_{Ht}Y_{SHt}$). To ensure a switch towards clean technologies in the polluting sector, the South needs to maintain tax on dirty innovation such that slightly more innovation occurs in clean than in dirty technologies as long as clean technologies remain less advanced than the dirty ones. From (21 and 22), at the tipping point ($s_{Sct} = s_{Sdt} = \frac{s_{SGt}}{2}$), the size of the intervention (T_t), relative to the size of the economy would be equal to:

$$\frac{T_t}{p_{Gt}Y_{SGt} + p_{Ht}Y_{SHt}} = \frac{\gamma (1-\gamma)\,\iota\kappa \left(\frac{s_{SGt}}{2}\right)^{\iota}}{\left(1+\kappa\left(\frac{s_{SGt}}{2}\right)^{\iota}\right)^{1-(\varepsilon-1)(1-\gamma)}} \frac{A_{Sdt-1}^{\varepsilon-1} - A_{Sct-1}^{\varepsilon-1}}{A_{Sct}^{\varepsilon-1} + A_{Sdt}^{\varepsilon-1}} \frac{p_{Gt}Y_{GSt}}{p_{Gt}Y_{SGt} + p_{Ht}Y_{SHt}} \quad (35)$$

For the switch to occur this ratio must remain smaller than g, otherwise, the gap between clean and dirty technologies in the South keeps increasing. The more the South specializes in the polluting sector G, the higher this ratio is as $\frac{p_{Gt}Y_{GXt}}{p_{Gt}Y_{SGt}+p_{Ht}Y_{SHt}}$ and s_{SGt} increase. Now, a tariff implemented by the North on the polluting good G can reduce the degree of specialization of the South in this sector, and therefore increases the chance that an intervention in the South becomes possible. An environmental disaster can then be avoided without the South specializing in the non-polluting sector.[46]

8.2 A.2 Worldwide entrepreneurs

So far I have assumed that innovation in the North and in the South only responded to local conditions. However many innovative firms are global and make their innovation decision based on the entire world market. I now study this case and, for simplicity, focus on the case where final consumption is Cobb-Douglas ($\sigma = 1$). I show that clean research subsidies alone can now prevent a disaster but that carbon taxes may still fail to do so. The conditions under which they would now refer to the relative size of the polluting sector in the South and the North rather than simply the pattern of comparative advantage.

More specifically, I consider that the producer of intermediate ι in sector $z \in \{c, d, H\}$ is the same in the North and in the South and his intermediate has the same productivity in both countries; however, intermediates are still not tradeable.[47] By hiring s_{Nzit} scientists in the North and s_{Szit} in the South, the entrepreneur for variety i in sector $z \in \{c, d, H\}$ with

[46] It is important to distinguish this case from one where the desire to participate or not in the South arises from a local government maximizing its own welfare. In that case, intervention in the South may be limited relative to the first best, both because the South social planner does not internalize the damage on the welfare of North citizens, and because of the leakage effect to the North which diminishes the efficiency of a South policy. However, being an exporter of good G pushes the South towards a relatively larger intervention to benefit from better terms of trade. A tariff on the polluting good would reduce these terms of trade effect and therefore potentially the willingness of the South to switch to clean technologies (see Copeland and Taylor (2005)). It would then be even more important to ensure that the tariff is directly related to emissions in the exporting country as mentioned in the previous paragraph.

[47] In this case trade balance typically does not hold since there will be income transfers across countries. Under free-trade this does not affect the pattern of production and emissions.

initial productivities $A_{zi(t-1)}$ can increases it to:

$$A_{zit} = \left(1 + \kappa \left(s^{\iota}_{Nzit} + s^{\iota}_{Szit}\right) \frac{A_{z(t-1)}^{\frac{1}{1-\gamma}}}{A_{zi(t-1)}^{\frac{1}{1-\gamma}}}\right)^{1-\gamma} A_{zi(t-1)}.$$

The results extend to the case where the innovation function is $\kappa \left(s_{Nzit} + s_{Szit}\right)^{\iota}$. As before, since profits are proportional to $(A_{zit}/A_{zt})^{\frac{1}{1-\gamma}}$, for $z \in \{c, d, H\}$, every entrepreneur hires the same number of scientists, and the law of motion of aggregate productivity can be written as:

$$A_{zt} = \left(1 + \kappa \left(s^{\iota}_{Xzt} + s^{\iota}_{(-X)zt}\right)\right)^{1-\gamma} A_{z(t-1)}.$$

In this subsection, the key to preventing a disaster will be to ensure that A_{ct} grows faster than A_{dt}, since this will bring the emission rate in the polluting sector down to zero even in the South. The next proposition shows that it is still the case that a carbon tax can fail at preventing an environmental disaster:

Proposition 7 *Assume that $\sigma = 1$, that clean technologies are sufficiently less advanced than dirty ones (A_{c0}/A_{d0} is sufficiently small), and that the South originally has a weakly larger market share than the North in the polluting good ($Y_{SG0} \geq Y_{NG0}$), then no carbon tax in the North can prevent a disaster, no matter how large the initial environmental quality $\overline{S}$ is.*

Proof. Appendix D.8 ■

Without direct research incentives, the innovation allocation is identical across countries. Within the polluting sector, the allocation of innovation across the clean and dirty subsectors favors the one with the largest revenues. In the North, a large carbon tax can ensure that the clean input subsector has nearly the same size as the entire North polluting sector, while in the South, if clean technologies are sufficiently less advanced (A_{c0}/A_{d0} is small), the size of the dirty subsector is close to the size of the South polluting sector. Worldwide, the size of the clean subsector is then close to the size of the polluting sector in the North, while the size of the dirty subsector is close to the size of the polluting sector in the South. If the South has a larger market share in the polluting sector, there would be more dirty than clean innovations, and A_{ct} would never catch up. When final consumption is Cobb-Douglas, the relative size of both countries in the production of the polluting good does not change with technologies, and, since a carbon tax can only increase the relative size of the South, an initially larger market share ensures that the South remains the biggest country in the polluting sector.[48]

[48] If the North uses a tax on dirty research on top of the carbon tax, the remark stays true but for Y_{NG0}/Y_{SG0} sufficiently small. In this case, the Northern social planner, can ensure that no scientists in the North innovate in dirty technologies. Yet, if Y_{NG0}/Y_{SG0} is small, most innovation in the North will occur in the non-polluting sector anyway, so most sector G innovation will occur in the South and will be determined by the South market, favoring dirty innovation. See Appendix D.8

If subsidies to clean research in the North are sufficiently strong, however, Northern scientists will nearly all innovate in clean intermediates, thereby also improving the productivity of clean intermediates in the South. In the meantime, innovation in the South will be prevented from moving fully towards sector G and dirty intermediates: even if the South fully specializes in sector G, entrepreneurs having monopoly rights in sector H will hire some scientists from the South to improve the productivity of the variety they own in the North. Therefore, a tariff (which is still part of the optimal policy) is not necessary to prevent a disaster, regardless of initial endowments.

Proposition 8 *Clean research subsidies in the North alone can prevent a disaster if the initial environmental quality is sufficiently large.*

As technological diffusion, the internationalization of the R&D process makes it possible for policy in the North to induce a switch to clean innovation in the South, so that the second best policy needs not feature a reversal of the pattern of comparative advantage. However, even with a very internationalized process, the North needs to undertake a very proactive policy towards the development of clean technologies; otherwise, the direction of innovation within sector G will be dictated by the economic conditions of the South instead of the North.

9 Appendix B: Main proofs of the paper

9.1 Appendix B.1 Characterization of the equilibrium in a given period

For this subsection I consider the economy at a given time, after innovation has occurred (I drop the subscript t for simplicity). To avoid repetition with the social optimum analysis, I do not impose that the economy is in laissez-faire: I let each country impose a subsidy $\widetilde{q}$ on all intermediates (it can differ across time or across countries without affecting anything as it has a pure scale effect) and a carbon tax τ_X. First I derive the aggregate production functions in each sector given prices, second I solve for prices and third I characterize the pattern of specialization in free trade. Finally, I derive the allocation of innovation.

9.1.1 B.1.1 Deriving aggregate production function.

First, note that if good G is produced, then both subsectors c and d are active. Assume that good G is produced in country X, the maximization problem for producers in subsector $z \in \{c, d\}$ leads to the demand function for capital and labor in assembly of good z:

$$r_X K_{Xz} = (1-\gamma)\,\alpha p_{Xz} Y_{Xz} \text{ and } w_X L_{Xz} = (1-\gamma)\,(1-\alpha)\, p_{Xz} Y_{Xz} \tag{36}$$

and the demand for intermediates:

$$\varphi_{Xzi} = \gamma p_{Xz} A_{Xzi} x_{zi}^{\gamma-1} \left(K_{Xz}^{\alpha} L_{Xz}^{1-\alpha}\right)^{1-\gamma}, \tag{37}$$

with φ_{Xzi} the consumer price of intermediate i. From 9, the cost of producing one unit of intermediate is given by $\psi\left(\frac{r_X}{\alpha}\right)^{\alpha}\left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}$. Monopolists maximizes profits by imposing a mark-up $1/\gamma$ on their costs, so that producer prices are given by:

$$\frac{\varphi_{Xzi}}{1-\widetilde{q}} = \frac{\psi}{\gamma}\left(\frac{r_X}{\alpha}\right)^{\alpha}\left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}. \tag{38}$$

The production of intermediates is then given by:

$$x_{Xzi} = \left(\frac{p_{Xz}\gamma^2}{(1-\widetilde{q})\,\psi}\left(\frac{\alpha}{r_X}\right)^{\alpha}\left(\frac{1-\alpha}{w_X}\right)^{1-\alpha}\right)^{\frac{1}{1-\gamma}} A_{Xzi}^{\frac{1}{1-\gamma}} K_{Xz}^{\alpha} L_{Xz}^{1-\alpha}, \tag{39}$$

and factor demands in the production of intermediate i in sector z follows:

$$K_{Xzi} = \left(\frac{\alpha}{r_X}\frac{w_X}{1-\alpha}\right)^{1-\alpha}\psi x_{Xzi} \text{ and } L_{Xzi} = \left(\frac{r_X}{\alpha}\frac{1-\alpha}{w_X}\right)^{\alpha}\psi x_{Xzi}. \tag{40}$$

Plugging in (36) and (39) into (8), I get the price of good z as:

$$p_{Xz} = \frac{1}{A_{Xz}}\frac{(1-\widetilde{q})^{\gamma}\,\psi^{\gamma}}{(1-\gamma)^{1-\gamma}\,\gamma^{2\gamma}}\left(\frac{r_X}{\alpha}\right)^{\alpha}\left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}, \tag{41}$$

where. Now, profit maximization by producers of good G leads to the demand function:

$$\frac{Y_{Xc}}{Y_{Xd}} = \left(\frac{p_{Xc}}{(1+\tau_X)\,p_{Xd}}\right)^{-\varepsilon}, \tag{42}$$

and the price of good G is given by $p_G = \left(p_{Xc}^{1-\varepsilon} + (1+\tau_X)^{1-\varepsilon}\,p_{Xd}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$, which, using (41), translates into:

$$p_{XG} = \frac{(1-\gamma)^{\gamma-1}\,\gamma^{-2\gamma}\,(1-\widetilde{q})^{\gamma}\,\psi}{\left(A_{Xz}^{\varepsilon-1} + \left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}\left(\frac{r_X}{\alpha}\right)^{\alpha}\left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}. \tag{43}$$

This relationship holds if country X produces good G, if country X does not produce good G, the equality is replaced by: $p_{XG} \leq \frac{(1-\gamma)^{\gamma-1}\gamma^{-2\gamma}(1-\widetilde{q})^{\gamma}\psi}{\left(A_{Xz}^{\varepsilon-1}+\left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}\left(\frac{r_X}{\alpha}\right)^{\alpha}\left(\frac{w_X}{1-\alpha}\right)^{1-\alpha}$. Similarly in sector H, $p_{XH} \leq \frac{(1-\gamma)^{\gamma-1}\gamma^{-2\gamma}(1-\widetilde{q})^{\gamma}\psi}{A_{XH}}\left(\frac{r_X}{\alpha}\right)^{\beta}\left(\frac{w_X}{1-\alpha}\right)^{1-\beta}$, with equality if good H is produced in country X.

Note that (42) gives:

$$Y_{Xd} = \left(\frac{(1+\tau_X)^{-1}A_{Xd}}{\left(A_{Xc}^{\varepsilon-1} + \left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}\right)^{\varepsilon} Y_{XG}, \tag{44}$$

which directly leads to the expression for the emission rate. Combining (36), (39), (40), (42), and (43), one gets that total factor employment in sector G satisfies:

$$K_{XG} = \left(\frac{\alpha}{r_X}\frac{w_X}{1-\alpha}\right)^{1-\alpha}\frac{1}{\zeta}\frac{A_{Xc}^{\varepsilon-1}+(1+\tau_X)^{-\varepsilon}A_{Xd}^{\varepsilon-1}}{\left(A_{Xc}^{\varepsilon-1}+\left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}}Y_{XG}, \tag{45}$$

$$L_{XG} = \left(\frac{r_X}{\alpha}\frac{1-\alpha}{w_X}\right)^{\alpha}\frac{1}{\zeta}\frac{A_{Xc}^{\varepsilon-1}+(1+\tau_X)^{-\varepsilon}A_{Xd}^{\varepsilon-1}}{\left(A_{Xc}^{\varepsilon-1}+\left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}}Y_{XG}, \tag{46}$$

with $\zeta \equiv \frac{\gamma^{2\gamma}(1-\gamma)^{1-\gamma}(1-\widetilde{q})^{1-\gamma}}{((1-\gamma)(1-\widetilde{q})+\gamma^2)\psi^{\gamma}}$. Combining these two expressions and following the same strategy in sector H, one gets:

$$Y_{XGt} = \zeta\frac{\left(A_{Xct}^{\varepsilon-1}+\left((1+\tau_{Xt})^{-1}A_{Xdt}\right)^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{A_{Xct}^{\varepsilon-1}+(1+\tau_{Xt})^{-\varepsilon}A_{Xdt}^{\varepsilon-1}}K_{XGt}^{\alpha}L_{XGt}^{1-\alpha} \text{ and } Y_{XHt} = \zeta A_{XHt}K_{XHt}^{\beta}L_{XHt}^{1-\beta}. \tag{47}$$

This equation translates into (19) when there is no carbon tax.

When both sectors are active, taking the ratio of (43) and the equivalent expression for p_{XH}, one can express the capital rent to wage ratio as

$$\frac{r_X}{w_X} = \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}}\left(\frac{\left(A_{Xc}^{\varepsilon-1}+\left((1+\tau_X)^{-1}A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{A_{XH}}\right)^{\frac{1}{\alpha-\beta}}\left(\frac{p_{XG}}{p_{XH}}\right)^{\frac{1}{\alpha-\beta}}.$$

Plugging this expression into (45) and (46) and the equivalent equations in sector H, and using factor market clearing (11), one gets a system of two equations with two unknowns (Y_{XG},Y_{XH}) that can be solved as:

$$\begin{aligned} Y_{XG} &= \frac{\zeta}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}}\frac{A_{XG}(\tau_X)}{1-\delta_X(\tau_X)} \\ &\times\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_X-\beta L_X\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\alpha}{\alpha-\beta}}\right) \end{aligned} \tag{48}$$

$$\begin{aligned} Y_{XH} &= \frac{\zeta}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}}A_{XH} \\ &\times\left(\alpha\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\beta}{\alpha-\beta}}L_X-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\beta}{\alpha-\beta}}K_X\right). \end{aligned} \tag{49}$$

where $A_{XG}(\tau_X) \equiv \left(A_{Xc}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$ is a measure of average productivity of sector G in country X, and $\delta_X(\tau_X) \equiv \frac{\tau_X A_{Xd}^{\varepsilon-1}(1+\tau_X)^{-\varepsilon}}{A_{Xc}^{\varepsilon-1}+(1+\tau_X)^{-(\varepsilon-1)}A_{Xd}^{\varepsilon-1}} \in [0,1)$ is a correction term (measuring the difference between imposing a tax and a decrease in productivity of the dirty input).

9.1.2 B.1.2 Equilibrium price

Consumer maximization leads to $\frac{p_{XG}}{p_{XH}} = \frac{\nu}{1-\nu}\left(\frac{C_{XH}}{C_{XG}}\right)^{\frac{1}{\sigma}}$. In autarky this translates into: $\frac{Y_{XG}}{Y_{XH}} = \left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{p_{XG}}{p_{XH}}\right)^{-\sigma}$, which combined with (48) and (49), defines the equilibrium autarky price uniquely (given technologies) since $\frac{Y_{XG}}{Y_{XH}}$ is increasing in $\frac{p_{XG}}{p_{XH}}$, and the right-hand side decreases. More specifically, one gets that the autarky price must satisfy:

$$\left(\frac{p_{XG}}{p_{XH}}\right)^{\sigma}\frac{A_{XG}(\tau_X)}{1-\delta_X(\tau_X)}\left(\begin{array}{c}\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_X \\ -\beta L_X\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\alpha}{\alpha-\beta}}\end{array}\right) \tag{50}$$

$$= \left(\frac{\nu}{1-\nu}\right)^{\sigma} A_{XH}\left(\begin{array}{c}\alpha\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{-\beta}{\alpha-\beta}}L_X \\ -\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_{XG}}{p_{XH}}\frac{A_{XG}(\tau_X)}{A_{XH}}\right)^{\frac{1-\beta}{\alpha-\beta}}K_X\end{array}\right)$$

If $\varepsilon A_{Xc}^{\varepsilon-1}+(1-(\varepsilon-1)\tau_X)(1+\tau_X)^{-\varepsilon}A_{Xd}^{\varepsilon-1} > 0$, $\frac{A_{XG}(\tau_X)}{1-\delta_X(\tau_X)}$ increases in A_{Xc} and always increase in A_{Xd} and decrease in τ_X. Therefore $\frac{p_{XG}}{p_{XH}}$ decreases in A_{Xc} (when $\varepsilon A_{Xc}^{\varepsilon-1}+(1-(\varepsilon-1)\tau_X)(1+\tau_X)^{-\varepsilon}A_{Xd}^{\varepsilon-1} > 0$), decreases in A_{Xd} and K_X and increases in A_{XH} and L_X. It is direct to check that in the absence of any tax, the relative autarky price of good G over good H is higher in the North than in the South if and only if $(A_{SG}/A_{SH})^{\frac{1}{\alpha-\beta}} K_S/L_S > (A_{NG}/A_{NH})^{\frac{1}{\alpha-\beta}} K_N/L_N$ (so that under free-trade the North imports good G in this case).

Under free-trade, the equilibrium price ratio is the same in both countries and satisfies

$$\frac{p_G}{p_H} = \frac{\nu}{1-\nu}\left(\frac{C_{XH}}{C_{XG}}\right)^{\frac{1}{\sigma}} = \frac{\nu}{1-\nu}\left(\frac{Y_{NH}+Y_{SH}}{Y_{NG}+Y_{SG}}\right)^{\frac{1}{\sigma}}, \tag{51}$$

which similarly defines uniquely the price ratio given technologies (as Y_{XH} is decreasing in the price ratio and Y_{XG} increasing). When both countries produce both goods, one can use (48) and (49) to get:

$$\left(\frac{p_G}{p_H}\right)^{\sigma}\left(\begin{array}{c}\frac{A_{NG}(\tau_N)}{1-\delta_N(\tau_N)}\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_N-\beta\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{-\alpha}{\alpha-\beta}}L_N\right)\\+\frac{A_{SG}(\tau_S)}{1-\delta_S(\tau S)}\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_S-\beta\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{-\alpha}{\alpha-\beta}}L_S\right)\end{array}\right) \tag{52}$$

$$= \left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\begin{array}{c}A_{NH}\left(\alpha\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{-\beta}{\alpha-\beta}}L_N-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_G}{p_H}\frac{A_{NG}(\tau_N)}{A_{NH}}\right)^{\frac{1-\beta}{\alpha-\beta}}K_N\right)\\+A_{SH}\left(\alpha\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{-\beta}{\alpha-\beta}}L_S-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_G}{p_H}\frac{A_{SG}(\tau_S)}{A_{SH}}\right)^{\frac{1-\beta}{\alpha-\beta}}K_S\right)\end{array}\right)$$

9.1.3 B.1.3 Pattern of specialization in free trade

I now derive the full pattern of specialization in free trade. To simplify expression I introduce the notations $\widetilde{K_X} \equiv \left(\frac{A_{XG}(\tau_X)^{1-\beta}}{A_{XH}^{1-\alpha}}\right)^{\frac{1}{\alpha-\beta}} K_X$ and $\widetilde{L_X} \equiv \left(\frac{A_{XH}^{\alpha}}{A_{XG}(\tau_X)^{\beta}}\right)^{\frac{1}{\alpha-\beta}} L_X$, which represent "effective endowments". Using (48), (49) and (52), assuming that both countries produce both goods, the condition $Y_{XG} > 0$ translates into

$$\frac{\widetilde{K_X}}{\widetilde{L_X}} > \frac{\beta}{1-\beta}\frac{\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}(1-\alpha)\left(\widetilde{K_N}+\widetilde{K_S}\right)+(1-\beta)\left(\frac{\widetilde{K_N}}{1-\delta_N}+\frac{\widetilde{K_S}}{1-\delta_S}\right)}{\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}\alpha\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\widetilde{L_N}+\widetilde{L_S}\right)+\beta\left(\frac{\widetilde{L_N}}{1-\delta_N}+\frac{\widetilde{L_S}}{1-\delta_S}\right)}, \tag{53}$$

and $Y_{XH} > 0$ into:

$$\frac{\widetilde{K_X}}{\widetilde{L_X}} < \frac{\alpha}{1-\alpha}\frac{\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}(1-\alpha)\left(\widetilde{K_N}+\widetilde{K_S}\right)+(1-\beta)\left(\frac{\widetilde{K_N}}{1-\delta_N}+\frac{\widetilde{K_S}}{1-\delta_S}\right)}{\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\left(\frac{\widetilde{L_X}}{\widetilde{K_X}}\right)^{\alpha-\beta}\right)^{1-\sigma}\alpha\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\widetilde{L_N}+\widetilde{L_S}\right)+\beta\left(\frac{\widetilde{L_N}}{1-\delta_N}+\frac{\widetilde{L_S}}{1-\delta_S}\right)}, \tag{54}$$

Therefore, conditions (53) and (54) define the set of endowments, productivity and taxes for which there is incomplete specialization in both countries.

Assume now that country X fully specializes in sector G, but country $-X$. does not fully specializes. Using (47), production of good G in X is given by: $Y_{XG} = \frac{\zeta}{1-\delta_X}\widetilde{K_X}^{\alpha}\widetilde{L_X}^{1-\alpha}$. Combining this expression with (51) and (48) and (49) for country $-X$, delivers an implicit equation for the price ratio, which can be used to that the condition $Y_{(-X)G} > 0$ is equivalent to:

$$\left(\frac{\widetilde{K_{-X}}}{\widetilde{L_{-X}}}\right)^{(\alpha-\beta)\sigma}\widetilde{K_{-X}}^{\beta}\widetilde{L_{-X}}^{1-\beta} > \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma}\frac{\widetilde{K_X}^{\alpha}\widetilde{L_X}^{1-\alpha}}{1-\delta_X}. \tag{55}$$

This case, therefore corresponds to the opposite of (53) and (55).

Similarly if country X specializes in sector H, one gets $Y_{XH}=\zeta\widetilde{K_X}^{\beta}\widetilde{L_X}^{1-\beta}$ and the condition $Y_{H(-X)}>0$ writes as:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma}\widetilde{K_X}^{\beta}\widetilde{L_X}^{1-\beta}<\left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{\widetilde{K_{-X}}}{\widetilde{L_{-X}}}\right)^{(\alpha-\beta)(1-\sigma)}\frac{\widetilde{K_{-X}}^{\beta}\widetilde{L_{-X}}^{1-\beta}}{1-\delta_{-X}}. \tag{56}$$

This case corresponds to the opposite of (54) and (56).

Finally the case where country X fully specializes in G while country $-X$ fully specializes in H corresponds to the opposite of (56).and the opposite of (55), for future use, it is convenient to express these two conditions with the actual endowments and productivities as:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma}A_{(-X)H}K_{-X}^{\beta}L_{-X}^{1-\beta}\geq\left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{K_X}{L_X}\right)^{(\alpha-\beta)(1-\sigma)}\frac{K_X^{\beta}L_X^{1-\beta}}{1-\delta_X}A_{XH}^{\sigma}\left(A_{XG}\left(\tau_X\right)\right)^{1-\sigma} \tag{57}$$

$$A_{(-X)H}^{1-\sigma}\left(A_{(-X)G}\left(\tau_{(-X)}\right)\right)^{\sigma}\left(\frac{L_{-X}}{K_{-X}}\right)^{(\alpha-\beta)(1-\sigma)}K_{-X}^{\alpha}L_{-X}^{1-\alpha}\leq\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\frac{1-\nu}{\nu}\right)^{\sigma}\frac{A_{XG}\left(\tau_X\right)K_X^{\alpha}L_X^{1-\alpha}}{1-\delta_X}. \tag{58}$$

One can show that these endowment sets have no overlap. Moreover, in each case scenario the relative price of good G over good H is uniquely defined, Therefore in free trade and for given technologies, the equilibrium is unique.

9.1.4 B.1.4 Equilibrium profits and innovation decision

Using (8), (39), I can express intermediates production in sector $z\in\{c,d\}$ as: $x_{Xzi}=\frac{p_{Xz}\gamma^2}{(1-\widetilde{q})\psi}\left(\frac{\alpha}{r_X}\right)^{\alpha}\left(\frac{1-\alpha}{w_X}\right)^{1-\alpha}\left(\frac{A_{Xzi}}{A_{Xz}}\right)^{\frac{1}{1-\gamma}}Y_{Xzt}$,combining this with (41) and (38) gives

$$\pi_{Xzit}=\frac{(1-\gamma)\gamma}{(1-\widetilde{q})}\left(\frac{A_{Xzit}}{A_{Xzt}}\right)^{\frac{1}{1-\gamma}}p_{Xzt}Y_{Xzt}, \tag{59}$$

or (21) when $\widetilde{q}=0$. Using (42), this translates into:

$$\pi_{Xcit}=\frac{\gamma(1-\gamma)}{(1-\widetilde{q})}\left(\frac{A_{Xcit}}{A_{Xct}}\right)^{\frac{1}{1-\gamma}}\frac{A_{Xct}^{\varepsilon-1}}{A_{cXt}^{\varepsilon-1}+\left((1+\tau_{Xt})^{-1}A_{dXt}\right)^{\varepsilon-1}}p_{Gt}Y_{GXt}, \tag{60}$$

$$\pi_{Xdit}=\frac{\gamma(1-\gamma)}{(1-\widetilde{q})}\left(\frac{A_{Xdit}}{A_{Xdt}}\right)^{\frac{1}{1-\gamma}}\frac{(1+\tau_{Xt})^{-\varepsilon}A_{Xdt}^{\varepsilon-1}}{A_{cXt}^{\varepsilon-1}+\left((1+\tau_{Xt})^{-1}A_{dXt}\right)^{\varepsilon-1}}p_{Gt}Y_{GXt}. \tag{61}$$

The same reasoning in sector H gives:

$$\pi_{XHit}=\frac{\gamma(1-\gamma)}{(1-\widetilde{q})}\left(\frac{A_{XHit}}{A_{XHt}}\right)^{\frac{1}{1-\gamma}}p_{XHt}Y_{XHt}. \tag{62}$$

To avoid repetition, I let both countries implement a tax q_{Xt} on the wages of scientists in the dirty subsector. Combining the first order conditions with respect to the number of scientists in the clean and dirty subsector (and assuming that some production takes place in sector G in country X) delivers the allocation of scientists within sector G as:

$$\frac{s_{Xct}^{1-\iota}\left(1+\kappa s_{Xct}^{\iota}\right)}{s_{Xdt}^{1-\iota}\left(1+\kappa s_{Xdt}^{\iota}\right)}=\frac{p_{Xct}Y_{Xct}}{p_{Xdt}Y_{Xdt}}=\frac{\left(1-q_{Xt}\right)\left(1+\tau_{Xt}\right)^{\varepsilon}A_{Xct}^{\varepsilon-1}}{A_{Xdt}^{\varepsilon-1}}, \tag{63}$$

where the second equality arises from (42) and (44). Similarly, combining the first order condition with respect to the number of scientists in sector H and subsector d, I get:

$$\frac{s_{Xdt}^{1-\iota}\left(1+\kappa s_{Xdt}^{\iota}\right)}{s_{XHt}^{1-\iota}\left(1+\kappa s_{XHt}^{\iota}\right)}=\frac{\left(1+\tau_{Xt}\right)^{-\varepsilon}A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+\left(\left(1+\tau_{Xt}\right)^{-1}A_{Xdt}\right)^{\varepsilon-1}}\frac{p_{XGt}Y_{XGt}}{p_{XHt}Y_{XHt}}. \tag{64}$$

With $\tau_{Xt}=0$, these two last equations write as (22) and (23).

9.2 Appendix B.2 Proofs of path dependence and of lemma 2

This proof requires several steps. First, I solve for the allocation of innovation within the polluting sector for a given mass of scientists. Second, I derive the comparative static of the growth rate of A_{XGt} and the incentive to innovate in sector G with the ratio of initial clean and dirty productivities for a given mass of scientists. Third, I show that under the hypothesis of the lemma, A_{SGt} grows faster than A_{NGt} and A_{NHt} grow faster than A_{SHt}. Forth I prove lemma 2.

9.2.1 Allocation of innovation within the polluting sector

To simplify notation, I introduce the function $\widetilde{\kappa}(s)\equiv\kappa s^{\iota}$, and the notation

$$a_{Xt}\equiv\min\left(\left(\frac{A_{Xc(t-1)}}{A_{Xd(t-1)}}\right)^{\varepsilon-1},\left(\frac{A_{Xd(t-1)}}{A_{Xc(t-1)}}\right)^{\varepsilon-1}\right)\leq 1.$$

In laissez-faire (22) leads to the following equality

$$\begin{aligned}&\widetilde{\kappa}'\left(s_A\left(a_{Xt},s_{XGt}\right)\right)\left(1+\widetilde{\kappa}\left(s_A\left(a_{Xt},s_{XGt}\right)\right)\right)^{(\varepsilon-1)(1-\gamma)-1}\\ ' \quad =\;&\widetilde{\kappa}'\left(s_a\left(a_{Xt},s_{XGt}\right)\right)\left(1+\widetilde{\kappa}\left(s_a\left(a_{Xt},s_{XGt}\right)\right)\right)^{(\varepsilon-1)(1-\gamma)-1}a_{Xt}\end{aligned}$$

where $s_A\left(a,s_G\right)$ is the allocation of scientists to the subsector amongst clean and dirty with the highest productivity level at $t-1$ and $s_a\left(a_X,s_{XGt}\right)$

9.2.2 Effect of relative productivity of clean and dirty on sector G growth and incentive to innovate

Denote $a_X \equiv \min\left(\left(\frac{A_{Xc}}{A_{Xd}}\right)^{\varepsilon-1}, \left(\frac{A_{Xd}}{A_{Xc}}\right)^{\varepsilon-1}\right) \leq 1$, and define

$$f(a, s_G) \equiv \frac{1+\widetilde{\kappa}(1-s_G)}{\widetilde{\kappa}'(1-s_G)}\frac{1}{2}\left(\frac{\widetilde{\kappa}'(s_a)(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)-1}a+\widetilde{\kappa}'(s_A)(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)-1}}{(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)}a+(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)}}\right),$$

where $s_A(a, s_G)$ and $s_a(a, s_G)$ are defined through and $s_A(a, s_G) + s_a(a, s_G) = s_G$, (that is s_A denotes the investments in the sector the most advanced between clean and dirty and s_a in the other sector). f represents for a given overall share of scientists in sector G, the ratio between the marginal benefit of an additional scientist in sector H divided by sector H revenues over the marginal benefit of an additional scientist in sector G divided by sector G's revenues. Note that with κ sufficiently small, $\widetilde{\kappa}'(s)(1+\widetilde{\kappa}(s))^{(\varepsilon-1)(1-\gamma)-1}$ is decreasing so that $s_a < s_A$. Next note that f is decreasing in s_G since $s_A(s_G)$ and $s_a(s_G)$ are both increasing in s_G. Rewriting $f(a, s_G) = \frac{(1+\widetilde{\kappa}(1-s_G))}{\widetilde{\kappa}'(1-s_G)}\frac{1}{\frac{1+\widetilde{\kappa}(s_a)}{\widetilde{\kappa}'(s_a)}+\frac{(1+\widetilde{\kappa}(s_A))}{\widetilde{\kappa}'(s_A)}}$, I get $\frac{\partial f}{\partial a} = \frac{(1-\iota)(1+\widetilde{\kappa}(1-s_G))}{\widetilde{\kappa}'(1-s_G)\iota\left(\frac{1+\widetilde{\kappa}(s_a)}{\widetilde{\kappa}'(s_a)}+\frac{(1+\widetilde{\kappa}(s_A))}{\widetilde{\kappa}'(s_A)}\right)^2}\left(\frac{1}{s_A^\iota}-\frac{1}{s_a^\iota}\right)\frac{\partial s_A}{\partial a} < 0$,so that f is decreasing in a, since $s_A > s_a$.

I define similarly the growth rate of sector G for a given number of scientists as: $g(a, s_G) \equiv \left(\frac{(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)}a+(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)}}{a+1}\right)$. g is increasing in s_G, and $\frac{\partial g}{\partial a} = \frac{(1+\widetilde{\kappa}(s_a))^{(\varepsilon-1)(1-\gamma)}-(1+\widetilde{\kappa}(s_A))^{(\varepsilon-1)(1-\gamma)}}{(a+1)^2} <$ 0,since $s_A > s_a$. Therefore for a given amount of scientists in sector G, the average productivity grows faster when the productivities of the two sectors are far from each other.

9.2.3 Growth rate of relative productivity

Here I prove the following lemma:

Lemma 4 *Assume that $\tau_{Xt} = 0$ for $X = N, S$ at all time, that country X initially has the comparative advantage in sector G $\left(\frac{A_{XG0}}{A_{XH0}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_X}{L_X} \geq \left(\frac{A_{(-X)G0}}{A_{(-X)H0}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_{(-X)}}{L_{(-X)}}$ and that (i) a_{X0} and $a_{(-X)0}$ are sufficiently small and the previous inequality is strict or (ii) $a_{X0} \leq a_{(-X)0}$; then at all points in time: $s_{XGt} \geq s_{(-X)Gt}$ (with a strict inequality if one of the previous equalities is strict), and, if one of the previous inequality is strict, A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} tend to infinity.*

Proof. To fix ideas, I assume that country X is the South, and that in both countries $A_{Xd0} > A_{Xc0}$. Assume that at time $t \geq 1$, I have $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S} \geq \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N}$, and either $a_{N(t-1)}, a_{S(t-1)}$ are both negligible (with a strict inequality) or $a_{S(t-1)} \leq a_{N(t-1)}$. Solving for entrepreneurs maximization (63) and (64), I get that in both countries,

$$\widetilde{\kappa}'(s_{Xct})(1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)-1}a_{X(t-1)} = \widetilde{\kappa}'(s_{Xdt})(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)-1},$$

$$f\left(a_{(t-1)}, s_{XGt}\right) = \frac{1+\widetilde{\kappa}\left(s_{XHt}\right)}{\widetilde{\kappa}'\left(s_{XHt}\right)}\frac{1}{2}\frac{\widetilde{\kappa}'\left(s_{Xct}\right)\left(1+\widetilde{\kappa}\left(s_{Xct}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}a_{Xt}+\widetilde{\kappa}'\left(s_{Xdt}\right)\left(1+\widetilde{\kappa}\left(s_{Xdt}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}}{\left(1+\widetilde{\kappa}\left(s_{Xct}\right)\right)^{(\varepsilon-1)(1-\gamma)}a_{Xt}+\left(1+\widetilde{\kappa}\left(s_{Xdt}\right)\right)^{(\varepsilon-1)(1-\gamma)}}$$
$$= \frac{p_{Ht}Y_{XHt}}{p_{Gt}Y_{XGt}}.$$

Using the expressions for (48) and (49), I get that -when there is not full specialization in any country- the equilibrium can be summarized by three equations:

$$f\left(a_{X(t-1)}, s_{XGt}\right)$$
$$= \frac{\alpha-(1-\alpha)\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\frac{p_{Gt}}{p_{Ht}}\frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)}A_{Xc(t-1)}^{\varepsilon-1}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)}A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma}A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_X}{L_X}}{(1-\beta)\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\frac{p_{Gt}}{p_{Ht}}\frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)}A_{Xc(t-1)}^{\varepsilon-1}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)}A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma}A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_X}{L_X}-\beta},$$

for $X \in \{N, S\}$ and the equation determining the price ratio $\frac{p_{Ht}}{p_{Gt}}$. ■

Now if $a_{N(t-1)} \geq a_{S(t-1)}$, then, at given s_{XGt}, the LHS of the previous equation is lower for the North than for the South. Similarly $\left(\frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)}A_{Xc(t-1)}^{\varepsilon-1}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)}A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma}A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_X}{L_X}$ can be rewritten as $\left(\frac{\left((1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)}a_{X(t-1)}+(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)}\right)^{\frac{1}{\varepsilon-1}}}{\left(a_{X(t-1)}+1\right)(1+\widetilde{\kappa}(s_{XHt}))^{1-\gamma}}\right)^{\frac{1}{(\alpha-\beta)}}\left(\frac{A_{XG(t-1)}}{A_{XH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_X}{L_X}$, which at given s_{XGt} is (weakly) higher for the South than for the North, since $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S} \geq \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N}$ and since g is decreasing in a, therefore at given s_{XGt}, the RHS of the previous equation is higher for the North than for the South. For given prices, both the LHS and the RHS are decreasing in s_{XGt}, but for sufficiently small κ, the LHS decreases faster, therefore $s_{SGt} \geq s_{NGt}$, with a strict inequality if either $a_{N(t-1)} > a_{S(t-1)}$ or if $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S} > \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N}$.

Similarly if both $a_{N(t-1)}$ and $a_{S(t-1)}$ are negligible (relative to the difference in comparative advantage), I get that $s_{Xdt} \simeq s_{XGt}$, $f\left(a_{(t-1)}, s_{XGt}\right) \simeq \frac{1+\widetilde{\kappa}(s_{XHt})}{\widetilde{\kappa}'(s_{XHt})}\frac{\widetilde{\kappa}'(s_{XGt})}{1+\widetilde{\kappa}(s_{XGt})}$ and

$$\frac{\left(\left(1+\widetilde{\kappa}\left(s_{Xct}\right)\right)^{(\varepsilon-1)(1-\gamma)}A_{Xc(t-1)}^{\varepsilon-1}+\left(1+\widetilde{\kappa}\left(s_{Xdt}\right)\right)^{(\varepsilon-1)(1-\gamma)}A_{Xd(t-1)}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{\left(1+\widetilde{\kappa}\left(s_{XHt}\right)\right)^{1-\gamma}A_{XH(t-1)}}$$
$$\simeq \frac{\left(1+\widetilde{\kappa}\left(s_{XGt}\right)\right)^{(1-\gamma)}A_{XG(t-1)}}{\left(1+\widetilde{\kappa}\left(s_{XHt}\right)\right)^{1-\gamma}A_{XH(t-1)}},$$

so that, following a similar reasoning, $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S} > \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N}$ leads to $s_{SGt} > s_{NGt}$.

Therefore,in both cases $A_{SGt}/A_{NGt} > A_{SG(t-1)}/A_{NG(t-1)}$ and $A_{NHt}/A_{SHt} > A_{NH(t-1)}/A_{SH(t-1)}$ (expect in the case where $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} = \left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ and $a_{N(t-1)} = a_{S(t-1)}$, where the strict inequalities are replaced by equalities). Note that $a_{Xt} < a_{X(t-1)}$, so if both $a_{N(t-1)}$ an $a_{S(t-1)}$ are negligible, a_{Nt} and a_{St} will be negligible too. Moreover, $\frac{1+\widetilde{\kappa}(s_A(a,s_G))}{1+\widetilde{\kappa}(s_a(a,s_G))}$ is increasing in s_G and decreasing in a, so if $a_{N(t-1)} \geq a_{S(t-1)}$ and $s_{SGt} \geq s_{NGt}$ then $a_{Nt} \geq a_{St}$.

The analysis extends directly to the case where one country specializes. By induction, this is enough to show that $s_{SGt} \geq s_{NGt}$ and, that A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} are increasing (with a strict inequality and strictly increasing if either $\left(\frac{A_{SG0}}{A_{SH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S} > \left(\frac{A_{NG0}}{A_{NH0}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ or $a_{N0} > a_{S0}$).

Nevertheless, having $s_{NGt} < s_{SGt}$ every period is not enough to conclude that A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} tend to infinity:s_{NGt} and s_{SGt} could converge towards each other. However this would require that either $\left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_S}{L_S}$ and $\left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}} \frac{K_N}{L_N}$ also converge towards each other (which is clearly ruled out), or that both s_{NGt} and s_{SGt} tend towards the same corner solution. Therefore, it should be the case that in both countries $\frac{p_{Ht}}{p_{Gt}} \frac{Y_{XHt}}{Y_{XGt}}$ either tends towards 0 or towards infinity, which is impossible too: in the Cobb-Douglas case $\frac{p_{Ht}}{p_{Gt}} \frac{Y_{NHt}+Y_{SHt}}{Y_{NGt}+Y_{SGt}} = \frac{1-\nu}{\nu}$, and when $\sigma < 1$ case, innovation overall favors the most backward sector preventing all scientists from innovating in the same sector in both countries asymptotically.

9.2.4 Reaching full specialization in finite time

Under the assumptions of lemma 2, using lemma 4, I get that at every period $s_{SGt} > s_{NGt}$ and that A_{SGt}/A_{NGt} and A_{NHt}/A_{SHt} grow unboundedly. Using the expressions (48) and (49), avoiding full specialization asymptotically requires that $\frac{p_{Ht}}{p_{Gt}} \frac{A_{NHt}}{A_{NGt}}$ remains bounded (from $Y_{NGt} \geq 0$) and similarly $\frac{p_{Gt}}{p_{Ht}} \frac{A_{SGt}}{A_{SHt}}$ remain bounded. Taking the product of the two, this leads towards $\frac{A_{NHt}}{A_{NGt}} \frac{A_{SGt}}{A_{SHt}}$ bounded which is a contradiction. Therefore at least one country fully specializes. For the sake of the argument assumes that the South fully specializes in sector G. If this is the case, note that asymptotically A_{SGt} must grow at the rate $(1+\kappa)^{1-\gamma} - 1$ (since eventually all scientists are in the dirty sector there). Then to avoid full specialization in the North in finite time, one must keep (from (55)):

$$\left(\frac{A_{NHt}}{A_{SGt}}\right)^{1-\sigma} K_N^{\alpha} L_N^{1-\alpha} > \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma} \left(\frac{1-\nu}{\nu}\right)^{\sigma} \left(\frac{A_{SGt}}{A_{NGt}}\right)^{\sigma} K_S^{\alpha} L_S^{1-\alpha},$$

where $0 < \sigma \leq 1$. Now A_{SGt}/A_{NGt} grows exponentially, while A_{NHt}/A_{SGt} cannot grow asymptotically since A_{SGt} asymptotically grows at the fastest rate. Therefore, keeping that inequality is impossible and the North must also fully specialize. Similarly if I had assumed that the North specialized first I would get that the South must also specialize in finite time. Therefore I have proved that I reach full specialization in finite time.

9.3 Appendix B.3 Proof of lemma 3

Since emission per-unit of the polluting good increases in the South, avoiding a disaster requires that the production of the polluting good stays bounded. If the South fully specializes in sector G avoiding a disaster is impossible, while if the South fully specializes in sector H, then in finite time all factors are allocated to the non-polluting sector by definition and the North exports the polluting good.

Assume now that there is not full specialization in the South, that is there are an infinite number of periods where the South produces both goods (and in the following I restrict attention to those periods). Rewriting (48) with $\tau_{St} = 0$ and $p_t = p_{SGt}/p_{Ht}$, production in sector G is given by

$$\begin{aligned} Y_{SGt} \quad = \quad & \frac{\zeta A_{SGt}}{(\alpha-\beta)} \left(\frac{\beta^{\beta\alpha} (1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha} (1-\alpha)^{(1-\alpha)\beta}} \right)^{\frac{1}{\alpha-\beta}} \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{-\alpha}{\alpha-\beta}} \\ & \times \left(\left(\frac{\alpha^{\alpha} (1-\alpha)^{(1-\alpha)}}{\beta^{\beta} (1-\beta)^{(1-\beta)}} \right)^{\frac{1}{\alpha-\beta}} (1-\beta) \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}} K_S - \beta L_S \right). \end{aligned} \tag{65}$$

Therefore to keep Y_{SGt} bounded it must be the case that $\left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}}$ is bounded, with either $\lim \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}} = \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \right)^{\frac{1}{\alpha-\beta}} \frac{\beta L_S}{(1-\beta)K_S}$ or with A_{SGt} bounded. Using (49), Y_{SHt} can be rewritten as:

$$\begin{aligned} Y_{SHt} \quad = \quad & \frac{\zeta A_{SHt}}{(\alpha-\beta)} \left(\frac{\beta^{\beta\alpha} (1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha} (1-\alpha)^{(1-\alpha)\beta}} \right)^{\frac{1}{\alpha-\beta}} \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{-\beta}{\alpha-\beta}} \\ & \times \left(\alpha L_S - \left(\frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha}}{\beta^{\beta} (1-\beta)^{(1-\beta)}} \right)^{\frac{1}{\alpha-\beta}} (1-\alpha) \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}} K_S \right). \end{aligned} \tag{66}$$

Combining these two expressions with (64) implies that the allocation of innovation in the South must satisfy

$$\frac{\kappa'(s_{Sdt})}{(1+\kappa(s_{Sdt}))} \frac{1+\kappa(1-s_{Sdt})}{\kappa'(1-s_{Sdt})} \frac{A_{Sdt}^{\varepsilon-1}}{A_{Sc0}^{\varepsilon-1} + A_{Sdt}^{\varepsilon-1}} = \frac{\alpha L_S - \left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}} \right)^{\frac{1}{\alpha-\beta}} (1-\alpha) \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}} K_S}{\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}} \right)^{\frac{1}{\alpha-\beta}} (1-\beta) \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}} K_S - \beta L_S}. \tag{67}$$

If $\lim \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}} \neq \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \right)^{\frac{1}{\alpha-\beta}} \frac{\beta L_S}{(1-\beta)K_S}$, s_{Sdt} cannot tend towards 0 in which case A_{SGt} would become unbounded, as demonstrated above this would lead to a disaster. Therefore it must be the case that $\lim \left(p_t \frac{A_{SGt}}{A_{SHt}} \right)^{\frac{1}{\alpha-\beta}} = \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \right)^{\frac{1}{\alpha-\beta}} \frac{\beta L_S}{(1-\beta)K_S}$ and so asymptotically all factors in the South (scientists, capital, labor) must be allocated to sector H.

Denoting M_{Gt} and M_{Ht} net imports from the North, (51) leads to:

$$\frac{p_{SGt}Y_{SGt}}{p_{SHt}Y_{SHt}} = \frac{\nu}{1-\nu}\left(\frac{Y_{SGt}-M_{Gt}}{Y_{SHt}-M_{Ht}}\right)^{-\frac{1}{\sigma}}\frac{Y_{SGt}}{Y_{SHt}}.$$

The right-hand side if greater than $\frac{\nu}{1-\nu}\left(\frac{Y_{SHt}}{Y_{SGt}}\right)^{\frac{1-\sigma}{\sigma}}$ if $M_{Gt}\geq 0$. But avoiding a disaster requires that the left-hand side tends towards 0, while $\frac{Y_{SHt}}{Y_{SGt}}$ becomes unbounded: this yields a contradiction, so the North must exports the polluting good G.

9.4 Appendix B.4 Proof of proposition 1

The proof is similar to the proof of the previous lemma: I show that as soon as a tax on dirty research or a carbon tax is implemented, $s_{NGt} < s_{SGt}$ and that this eventually leads to full specialization. Note that in the presence of a carbon tax and a tax on dirty research, the allocation of research in the North now obeys:

$$\begin{aligned}&\frac{\left(1+\widetilde{\kappa}\left(s_{NHt}\right)\right)\left(\begin{array}{c}\widetilde{\kappa}'\left(s_{Nct}\right)\left(1+\widetilde{\kappa}\left(s_{Nct}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}A_{Nc(t-1)}^{\varepsilon-1}\\+\widetilde{\kappa}'\left(s_{Ndt}\right)\left(1-q_t\right)\left(1+\tau_{Nt}\right)^{-\varepsilon}\left(1+\widetilde{\kappa}\left(s_{Ndt}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}A_{Nd(t-1)}^{\varepsilon-1}\end{array}\right)}{\widetilde{\kappa}'\left(s_{NHt}\right)2\left(\left(1+\widetilde{\kappa}\left(s_{Nct}\right)\right)^{(\varepsilon-1)(1-\gamma)}A_{Nc(t-1)}^{\varepsilon-1}+\left(1+\tau_{Nt}\right)^{1-\varepsilon}\left(1+\widetilde{\kappa}\left(s_{Ndt}\right)\right)^{(\varepsilon-1)(1-\gamma)}A_{Nd(t-1)}^{\varepsilon-1}\right)} \quad (68)\\ = \; &\frac{p_{Ht}Y_{NHt}}{p_{Gt}Y_{NGt}},\end{aligned}$$

with:

$$\widetilde{\kappa}'\left(s_{Nct}\right)\left(1+\widetilde{\kappa}\left(s_{Nct}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}A_{Nc(t-1)} = \widetilde{\kappa}'\left(s_{Ndt}\right)\left(1+\widetilde{\kappa}\left(s_{Ndt}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}\left(1-q_t\right)\left(1+\tau_{Nt}\right)^{1-\varepsilon}A_{Nd(t-1)}$$

and, following (48) and (49), $\frac{p_{Ht}Y_{NHt}}{p_{Gt}Y_{NGt}}$ is increasing in τ at given prices and technological levels. Without loss of generality, assume that from the first period a tax on dirty research or a carbon tax is implemented.

First I explain how the presence the tax on dirty research affects the incentive to innovate in sector G, second I show that in the first period ($t=1$), it is necessarily the case that $s_{NG1} < s_{SG1}$, third I show that in all following periods the same logic applies and finally I show that full specialization is reached.

9.4.1 Tax on dirty research and incentive to innovate

Define

$$\begin{aligned}&f\left(a_{N(t-1)}, s_{NGt}, 1-q_t\right)\\ \equiv\; &\frac{1+\widetilde{\kappa}\left(s_{NHt}\right)}{\widetilde{\kappa}'\left(s_{NHt}\right)}\frac{1}{2}\frac{\widetilde{\kappa}'\left(s_{Nct}\right)\left(1+\widetilde{\kappa}\left(s_{Nct}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}a_{N(t-1)}+\left(1-q_t\right)\widetilde{\kappa}'\left(s_{Ndt}\right)\left(1+\widetilde{\kappa}\left(s_{Ndt}\right)\right)^{(\varepsilon-1)(1-\gamma)-1}}{\left(1+\widetilde{\kappa}\left(s_{Nct}\right)\right)^{(\varepsilon-1)(1-\gamma)}a_{N(t-1)}+\left(1+\widetilde{\kappa}\left(s_{Ndt}\right)\right)^{(\varepsilon-1)(1-\gamma)}}\end{aligned}$$

with s_{Nct} and s_{Ndt} defined by

$$\widetilde{\kappa}'(s_{Nct})(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1} a_{Nct} = (1-q_t)\widetilde{\kappa}'(s_{Ndt})(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)-1}.$$

(Note I keep the same notation as above but here $a_{N(t-1)} = \frac{(1+\tau_{Nt})A_{Nc(t-1)}}{A_{Nd(t-1)}}$ and can be greater or smaller than 1).

It is direct to show that $\widetilde{\kappa}'(s_{Nct})(1+\widetilde{\kappa}(s_{Nct}))^{(\varepsilon-1)(1-\gamma)-1} a_{N(t-1)} + (1-q_t)\widetilde{\kappa}'(s_{Ndt})(1+\widetilde{\kappa}(s_{Ndt}))^{(\varepsilon-1)(1-\gamma)-1}$ decreases with q_t when κ is sufficiently small to ensure that $\widetilde{\kappa}'(s)(1+\widetilde{\kappa}(s))^{(\varepsilon-1)(1-\gamma)-1}$ is decreasing, which has been assumed so far. The denominator is decreasing with q_t since for a given mass scientists in sector G, $q_t = 0$, maximizes the growth rate of average productivity. However for sufficiently small κ, the variations in the denominator are negligible, and for $q_t > 0$, $f\left(a_{N(t-1)}, s_{NGt}, 1-q_t\right) < f\left(a_{N(t-1)}, s_{NGt}, 1\right) = f\left(a_{N(t-1)}, s_{NGt}\right)$.

9.4.2 Showing that $s_{NG1} < s_{SG1}$

The equilibrium allocation of innovation in the North (68) can then be rewritten as

$$f\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, s_{NG1}, \frac{1-q_1}{1+\tau_{N1}}\right) = \frac{p_{H1}Y_{NH1}}{p_{G1}Y_{NG1}}.$$

Since $f\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, s_{NG1}, \frac{1-q_1}{1+\tau_{N1}}\right) < f\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, s_{NG1}\right)$, and that the tax increases $\frac{p_{Ht}Y_{NHt}}{p_{Gt}Y_{NGt}}$, the logic of the proof of lemma 2 would fully apply provided that

$$\min\left((1+\tau_{N1})\frac{A_{Nc0}}{A_{Nd0}}, (1+\tau_{N1})^{-1}\frac{A_{Nd0}}{A_{Nc0}}\right) \geq \frac{A_{Sc0}}{A_{Sd0}},$$

since $\frac{A_{Nc0}}{A_{Nd0}} \geq \frac{A_{Sc0}}{A_{Sd0}}$, this is necessarily satisfies unless $(1+\tau_{N1}) \geq \frac{A_{Nd0}}{A_{Nc0}}\frac{A_{Sd0}}{A_{Sc0}}$. However, for $\frac{A_{Sc0}}{A_{Sd0}}$ sufficiently small, the corresponding tax will be very large, so that the difference in initial comparative advantage between the North and the South would become large, and following the logic of lemma 2 - but with a_{Sc0} negligible relative to the difference in comparative advantage-, in that case too $s_{NG1} < s_{SG1}$. The presence of both the tax on dirty research and the carbon tax distorts innovation such that A_{NG1} grows less than without the tax. As a consequence, $\left(\frac{A_{SG1}}{A_{SH1}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S} \geq \left(\frac{A_{NG1}}{A_{NH1}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N}$.

9.4.3 Showing that $s_{NGt} < s_{SGt}$

As long as $A_{Nd(t-1)} \geq A_{Nc(t-1)}$, the same logic exactly applies: every period $\frac{A_{Nc(t-1)}}{A_{Nd(t-1)}} \geq \frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$, since less scientists are allocated to sector G in the North and the allocation is tilted towards the clean subsector. Once $A_{Nc(t-1)} \geq A_{Nd(t-1)}$, then $\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} = \min\left(\frac{A_{Nc(t-1)}}{A_{Nd(t-1)}}, \frac{A_{Nd(t-1)}}{A_{Nc(t-1)}}\right)$ could in principle decrease faster than $\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$ since all scientists in sector G can be allocated to the clean subsector in the North whereas they are shared between the two sectors in the

South; with $\frac{A_{Sc0}}{A_{Sd0}}$ sufficiently small, and $s_{NG(t-1)} < s_{SG(t-1)}$, nearly all scientists in the South can be allocated to the dirty subsector so that this cannot happen and $\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$ decreases faster than $\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}}$. It is still possible to get $(1+\tau_{Nt})^{-1}\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} \leq \frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$, with a sufficient large tax (the required tax being increasing over time since $\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$ decreases faster than $\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}}$), this tax would then again lead to a large difference in comparative advantage, unless $A_{Nd(t-1)}$ has become sufficiently small relative to $A_{Nc(t-1)}$, but this will not happen before a significant number of periods, by which the difference in comparative advantage - which will have built up with innovation - will be large. Therefore $s_{SGt} > s_{NGt}$ every period.

9.4.4 Reaching full specialization

Therefore here as well, $\frac{A_{SGt}}{A_{NGt}}$ and $\frac{A_{NHt}}{A_{SHt}}$ grow unboundedly. From (48) and (49), this necessarily to specialization in at least one country. Assume that there is full specialization in sector G in the South, so that asymptotically A_{SGt} must grow at the rate $(1+\kappa)^{1-\gamma}-1$. Then to avoid full specialization in the North in finite time, I need to keep (from (58)):

$$A_{NHt}^{1-\sigma}\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{\sigma}{\varepsilon-1}}K_N^{\alpha}L_N^{1-\alpha} \geq \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma}A_{SGt}K_X^{\alpha}L_X^{1-\alpha},$$

which here again is impossible. Similarly if the North fully specializes in sector H, avoiding specialization in the South would also be impossible. Therefore both countries fully specialize, the emissions in the South necessarily grow unbounded and there is a disaster.

9.5 Appendix B.5 Proofs of subsection 3.4

9.5.1 B.5.1 Proof of proposition 2

If the North implements a tariff that prohibits any trade, innovation in the South will be balanced between dirty intermediates and sector H. Using sufficiently large clean research subsidies, the social planner can allocate nearly all innovation in the North towards clean intermediates. Therefore, after a finite number of period, I will have: $\left(\frac{A_{NG(t-1)}}{A_{NH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_N}{L_N} > \left(\frac{A_{SG(t-1)}}{A_{SH(t-1)}}\right)^{\frac{1}{\alpha-\beta}}\frac{K_S}{L_S}$ and $a_{N(t-1)} = \frac{A_{Nd(t-1)}}{A_{Nc(t-1)}} < \frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$. Using lemma 2, the North will specialize in sector G while the South will specialize in H. Worldwide emissions are bounded, so for a sufficiently large $\overline{S}$, a disaster is avoided.

9.5.2 B.5.2 Proof of remark 2

Assume that at some period $(t-1)$:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)A_{NH(t-1)}K_N^{\beta}L_N^{1-\beta} > (1+m)\left(\frac{1-\nu}{\nu}\right)K_S^{\beta}L_S^{1-\beta}A_{SH(t-1)}, \tag{69}$$

$$A_{NG(t-1)}K_N^{\alpha}L_N^{1-\alpha}(1+m) < \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)\left(\frac{1-\nu}{\nu}\right)A_{SG(t-1)}K_S^{\alpha}L_S^{1-\alpha}, \tag{70}$$

where $m>0$, is sufficiently large that regardless of the allocation of research in the following period, (57) and (58) are satisfied (with $\tau_S = 0$ and $\sigma = 1$). Then the North fully specializes in H and the South in G, therefore, all scientists in the South are allocated to sector G. Now, as innovation in the South occurs in the polluting sector A_{Sct}/A_{Sdt} becomes arbitrarily small. Taking a first order approximation, the number of scientists allocated to the clean sector in the South will be given by: $s_c = \left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}$. Therefore, the growth rate of A_{Sdt} is $(1+\kappa)^{1-\gamma}\left(1-\frac{(1-\gamma)\iota}{1+\kappa}\left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}\right)-1$, while with all scientists allocated to clean technologies in the North, the growth rate of A_{Nct} is $(1+\kappa)^{1-\gamma}-1$. Since the series $\sum -\frac{(1-\gamma)\iota}{1+\kappa}\left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}$ is converging, then $\prod_t (1-\kappa)^{1-\gamma}\left(1-\frac{(1-\gamma)\iota}{1+\kappa}\left(\frac{A_{ct-1}^{\varepsilon-1}}{A_{dt-1}^{\varepsilon-1}}\right)^{\frac{1}{1-\iota}}\right)$ is not dominated by $\prod_t (1+\kappa)^{1-\gamma}$, in other words, A_{Sdt} can remain larger than MA_{NGt} where M is a constant. In other words, (69) and (70) remain satisfied in the next period.

9.6 Appendix B.6 Proof of Proposition 3

Here I solve first for the problem of maximizing (1), I denote the Lagrange parameters (with the corresponding constraints in parentheses): λ_{Xt} (3), λ_{XHt} (4), λ_{XGt} (7), λ_{Xzt} (8), φ_{Xzit} (9), φ_{XHit} (5), η_{XKt} (11) for capital, η_{XLt} (11) for labor, θ_{Gt} (12) in sector G, θ_{Ht} (12) in sector H, ω_t (16), μ_{Xzit} (13), υ_{Xt} (15), in addition the social planner faces the constraints: $0 \le Y_{XGt}$ and $0 \le Y_{XHt}$, with Lagrange parameters: ι_{XGt}, ι_{XHt}. Taking the first order condition with respect to Y_{XHt} and Y_{XGt} gives:

$$\lambda_{XHt} = \theta_{Ht} + \iota_{XHt} \text{ and } \lambda_{XGt} = \theta_{Gt} + \iota_{XGt}.$$

Defining $u(C_{Wt}, S_t) \equiv \frac{(\nu(S_t)C_{Wt})^{1-\eta}}{1-\eta}$ with $C_{Wt} \equiv C_{Nt} + C_{St}$, first order conditions with respect to C_{Nt} and C_{St} lead to:

$$\frac{1}{(1+\rho)^t}\frac{\partial u}{\partial C}(C_{Wt}, S_t) = \frac{\nu(S_t)^{1-\eta}}{(1+\rho)^t}C_{Wt}^{-\eta} = \lambda_{Xt} \equiv \lambda_t.$$

First order conditions with respect to C_{XGt} and C_{XHt} give:

$$\lambda_t \nu C_{XGt}^{\frac{-1}{\sigma}}\left(\nu C_{XGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu)C_{XHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \theta_{Gt}, \tag{71}$$

$$\lambda_t (1-\nu) C_{XHt}^{\frac{-1}{\sigma}}\left(\nu C_{XGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu)C_{XHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \theta_{Ht}. \tag{72}$$

θ_{Gt}/λ_t and θ_{Ht}/λ_t can be interpreted as consumer prices in terms of units of welfare. To emphasize this interpretation, I denote $\widehat{p}_{Gt} = \theta_{Gt}/\lambda_t$ and $\widehat{p}_{Ht} = \theta_{Gt}/\lambda_t$. I then get:

$$\frac{\widehat{p}_{Gt}}{\widehat{p}_{Ht}} = \frac{\nu}{1-\nu}\left(\frac{C_{XHt}}{C_{XGt}}\right)^{\frac{1}{\sigma}} = \frac{\nu}{1-\nu}\left(\frac{C_{NHt}+C_{SHt}}{C_{NGt}+C_{SGt}}\right)^{\frac{1}{\sigma}},$$

which is the equivalent to the equilibrium condition (51). Taking the first order condition with respect to with respect to Y_{XHt} and Y_{XGt} gives:

$$\lambda_{XHt} = \theta_{Ht} + \iota_{XHt} \text{ and } \lambda_{XGt} = \theta_{Gt} + \iota_{XGt},$$

so that when production of good $Y \in \{G, H\}$ takes place: $\lambda_{XYt} = \theta_{Yt}$. Defining $\widehat{\varphi}_{Xzit} \equiv \frac{\varphi_{Xzit}}{\lambda_t}$ and $\widehat{p}_{Xzt} \equiv \frac{\lambda_{Xzt}}{\lambda_t}$, which can be interpreted as the price of intermediate x_{Xzi} and of input Y_{Xz}, first order condition with respect to x_{Xzit} gives:

$$\widehat{\varphi}_{Xzit} = \gamma \widehat{p}_{Xzt} A_{Xzit} x_{zit}^{\gamma-1}\left(K_{Xzt}^{\alpha} L_{Xzt}^{1-\alpha}\right)^{1-\gamma},$$

which is the same as (37). Combining the first order conditions with respect to K_{Xzit} and L_{Xzit} further gives

$$\widehat{\varphi}_{Xzit} = \frac{\psi \widehat{r}_{XHt}^{\alpha} \widehat{w}_{XLt}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}},$$

where $\widehat{r}_{XHt} \equiv \frac{\eta_{XKt}}{\lambda_t}$ and $\widehat{w}_{XLt} \equiv \frac{\eta_{XLt}}{\lambda_t}$ are the prices of capital and labor in country X. This last equation is identical to (38) when the optimal subsidy $\widetilde{q} = 1-\gamma$ is used. Recovering the equations equivalent to (40) is direct. First order conditions with respect to K_{Xzt} and L_{Xzt} allow to recover the equations equivalent to (36). Now taking the first order condition with respect to Y_{Xdt} and Y_{Xct}, one gets (when $Y_{XGt} \neq 0$):

$$\begin{aligned}
\widehat{p}_{Gt} Y_{Xdt}^{-\frac{1}{\varepsilon}}\left(Y_{Xct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{Xdt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} &= \widehat{p}_{Xdt} + \xi\frac{\omega_t}{\lambda_t}, \\
\widehat{p}_{Gt} Y_{Xct}^{-\frac{1}{\varepsilon}}\left(Y_{Xct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{Xdt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} &= \widehat{p}_{Xct}
\end{aligned}$$

this is equivalent to (42) with a tax

$$\tau_t = \xi\frac{\omega_t}{\lambda_{Xdt}} = \xi\frac{(1+\rho)^t \omega_t}{\widehat{p}_{Xdt}\frac{\partial u}{\partial C}(C_{Wt}, S_t)}. \tag{73}$$

Therefore:

$$\lambda_{XG} = \frac{\psi^{\gamma} \eta_{XKt}^{\alpha} \eta_{XLt}^{1-\alpha}}{\left(A_{Xz}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} (1-\gamma)^{1-\gamma}\gamma^{\gamma}\alpha^{\alpha}(1-\alpha)^{1-\alpha}},$$

so that, as in equilibrium, country X specializes in good H if

$$\widehat{p}_{XG} < \frac{\psi^{\gamma}\widehat{r}_{Xt}^{\alpha}\widehat{w}_{Xt}^{1-\alpha}}{\left(A_{Xz}^{\varepsilon-1} + \left((1+\tau_X)^{-1} A_{Xd}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} (1-\gamma)^{1-\gamma}\gamma^{\gamma}\alpha^{\alpha}(1-\alpha)^{1-\alpha}}.$$

The analysis of sector H is identical except that there is of course no tax there.

Because of the environmental dynamic equation for S_t, the quality of the environment will never reach back $\overline{S}$ in finite time, so it will remain below this bound. Taking the first order condition with respect to S_t (and taking into account that if $S_t = 0$, one gets $S_{t+1} = S_t = 0$) gives:

$$\omega_t = \frac{1}{(1+\rho)^t}\frac{\partial u}{\partial S}(C_{Ns} + C_{Ss}, S_s) + (1 - I_{S_t>0}\Delta)\,\omega_{t+1}, \tag{74}$$

which achieves to describe the optimal tax.

I now turn to the optimal solution for the innovation part. First as in the equilibrium case, only the average level of technologies (defined in (14)) matter, since the law of motion can be written as

$$A_{Xzit}^{\frac{1}{1-\gamma}} = A_{Xzi(t-1)}^{\frac{1}{1-\gamma}} + \widetilde{\kappa}(s_{Xzit})\,A_{Xz(t-1)}^{\frac{1}{1-\gamma}}, \text{ for } z \in \{c, d, H\}, \tag{75}$$

the solution is also symmetric: $s_{Xzit} = s_{Xzt}$ for $z \in \{c, d, H\}$.

Now taking the first order condition with respect to A_{Xzt}, gives:

$$\begin{aligned}
& \mu_{Xzit} \\
= \; & \lambda_{XHt} x_{Xzit}^{\gamma}\left(K_{Xzt}^{\overline{\alpha\beta}} L_{Xzt}^{1-\overline{\alpha\beta}}\right)^{1-\gamma} \\
& + \mu_{Nzi(t+1)}\left(\left(1 + \widetilde{\kappa}(s_{Xzt})\left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{1}{1-\gamma}}\right)^{1-\gamma} - \widetilde{\kappa}(s_{Xzt})\left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{1}{1-\gamma}}\left(1 + \widetilde{\kappa}(s_{Xzt})\left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma}\right) \\
& + \int_0^1 \widetilde{\kappa}(s_{Xzt})\frac{A_{Xzit}^{\frac{1}{1-\gamma}-1}}{A_{Xzjt}^{\frac{1}{1-\gamma}}}\left(1 + \widetilde{\kappa}(s_{Xzt})\left(\frac{A_{Xzt}}{A_{Xzjt}}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma} A_{Xzjt}\mu_{Xzj(t+1)}dj,
\end{aligned}$$

(with $\overline{\alpha\beta} = \alpha$ if $z \in \{c, d\}$ and $\overline{\alpha\beta} = \beta$ if $z = H$), multiplying both sides by $A_{Xzit}^{\frac{-\gamma}{1-\gamma}}$, one gets:

$$\mu_{Xzit}A_{Xzit}^{\frac{-\gamma}{1-\gamma}} = \lambda_{Xzt}x_{Xzit}^{\gamma}A_{Xzit}^{\frac{-\gamma}{1-\gamma}}\left(K_{Xzt}^{\beta}L_{Xzt}^{1-\beta}\right)^{1-\gamma} + \mu_{Nzi(t+1)}A_{Xzit+1}^{\frac{-\gamma}{1-\gamma}} + \widetilde{\kappa}(s_{Xzt})\int A_{Xzjt+1}^{\frac{-\gamma}{1-\gamma}}\mu_{Nzj(t+1)}dj,$$

since the equivalent of (39) also holds for sector H, $\lambda_{Xzt}x_{Xzit}^{\gamma}A_{Xzit}^{\frac{-\gamma}{1-\gamma}}\left(K_{Xzt}^{\overline{\alpha\beta}}L_{Xzt}^{1-\overline{\alpha\beta}}\right)^{1-\gamma}$ is a constant across varieties i. Therefore, $\mu_{Xzit}A_{Xzit}^{\frac{-\gamma}{1-\gamma}}$ is constant across varieties and one can define:

$$\mu_{Xzt} \equiv \left(\frac{A_{Xzt}}{A_{Xzit}}\right)^{\frac{\gamma}{1-\gamma}}\mu_{Xzit},$$

which represents the shadow value of one unit of average productivity in sector z, in country X at time t. I can then show that μ_{Xzt} follows the law of motion:

$$\mu_{Xzt}A_{Xzt} = \lambda_{Xzt}Y_{Xzt} + \mu_{Xz(t+1)}A_{Xzt+1}. \tag{76}$$

Now taking the first order condition with respect to s_{Xzit} one gets:

$$\nu_{Xt} = \mu_{Xzit}\left(1-\gamma\right)\widetilde{\kappa}'\left(s_{Xzit}\right)\left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}}\right)^{\frac{1}{1-\gamma}}\left(1+\widetilde{\kappa}\left(s_{Xzit}\right)\left(\frac{A_{Xz(t-1)}}{A_{Xzi(t-1)}}\right)^{\frac{1}{1-\gamma}}\right)^{-\gamma}A_{Xzi(t-1)},$$

which can then be rewritten as:

$$\nu_{Xt} = \frac{\left(1-\gamma\right)\widetilde{\kappa}'\left(s_{Xzt}\right)}{1+\widetilde{\kappa}\left(s_{Xzt}\right)}\mu_{Xzt}A_{Xzt}.$$

Or defining $\widehat{\nu}_{Xt} = \nu_{Xt}/\lambda_{Xt}$, the wage of scientists in terms of utility units, I can rewrite the last equality as

$$\widehat{\nu}_{Xt} = \frac{\left(1-\gamma\right)\widetilde{\kappa}'\left(s_{Xzt}\right)}{1+\widetilde{\kappa}\left(s_{Xzt}\right)}\sum_{s=t}^{\infty}\frac{\lambda_s}{\lambda_t}\widehat{p}_{zs}Y_{zs}. \tag{77}$$

Using (41), (42), (43), (44) gives:

$$\frac{p_{Xct}Y_{Xct}}{p_{XGt}Y_{XGt}} = \frac{A_{Xct}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+\left(\left(1+\tau_{Xt}\right)^{-1}A_{Xdt}\right)^{\varepsilon-1}},\quad \frac{p_{Xdt}Y_{Xdt}}{p_{XGt}Y_{XGt}} = \frac{\left(1+\tau_{Xt}\right)^{-\varepsilon}A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+\left(\left(1+\tau_{Xt}\right)^{-1}A_{Xdt}\right)^{\varepsilon-1}}.$$

Combining this last two equations with (77) and (76), I get (25).

Solving for the maximization of (2) can be done in a very similar way. One gets:

$$\begin{aligned}\lambda_t &= \frac{\nu\left(S_t\right)^{1-\eta}}{\left(1+\rho\right)^t}\Psi C_{Nt}^{-\eta} = \frac{\nu\left(S_t\right)^{1-\eta}}{\left(1+\rho\right)^t}\left(1-\Psi\right)C_{St}^{-\eta}\\ &= \frac{1}{\left(1+\rho\right)^t}\left(\Psi^{\frac{1}{\eta}}+\left(1-\Psi\right)^{\frac{1}{\eta}}\right)^{\eta}\frac{\partial u}{\partial C}\left(C_{Wt},S_t\right),\end{aligned}$$

and all results carry through provided that one replaces u by $\left(\Psi^{\frac{1}{\eta}}+\left(1-\Psi\right)^{\frac{1}{\eta}}\right)^{\eta}u$ (note that this does not affect the value of the optimal tax or the optimal allocation of scientists).

9.7 Appendix B.7 Proof of remark 3

First, I show that the social planner always chooses to avoid a disaster. Avoiding a disaster is always feasible since the social planner could simply stop production of dirty inputs in both countries. If $\eta \geq 1$ a disaster leads to $U = -\infty$, so the social planner would avoid it. If $\eta < 1$, a disaster leads to a utility flow equal to 0 in the current and all the following periods, whereas by reducing the production of dirty input enough, the social planner can achieve a positive utility flow, without any cost on previous periods.

The rest of the proof has three steps: first I show that when there is no environmental damage, and $\eta \leq 1$, the social planner maximizes long-run growth, second I derive the long-run growth properties of the switch and full specialization in finite time case, third I conclude.

9.7.1 Maximizing long-run growth

Here, I establish the following lemma:

Lemma 5 *Assume $\eta \leq 1$. Consider an allocation (C_{Wt}, S_t) such that there is a t_1, a $A > 0$ and a $g > 0$, such that for $t > t_1$, $S_t = \overline{S}$ and $\lim C_{Wt}/(1+g)^t = A > 0$. Then for ρ sufficiently small the social planner would rather choose the path (C_{Wt}, S_t) to any other (C'_{Wt}, S'_t) if there is a $M > 1$, such that for $t > t_2$, $C'_{Wt} < \frac{1}{M} C_{Wt}$.*

Then there is a t_3 such that for all $t > t_3$, $C'_{Wt} < \frac{1}{M} C_{Wt}$, $S_t = \overline{S}$ and there is a ε such that $C_{Wt} > (A - \varepsilon)(1+g)^t$.

First I consider the case where $\eta = 1$:

$$\begin{aligned} & U - U' \\ = \;& \sum_{t=0}^{t_3-1} \frac{\log(C_{Wt}) - \log(C'_{Wt}) + \log\nu(S_t) - \log\nu(S'_t)}{(1+\rho)^t} + \sum_{t=t_3}^{\infty} \frac{\log(C_{Wt}) - \log(C'_{Wt})}{(1+\rho)^t} + \sum_{t=t_3}^{\infty} \frac{\log\nu\left(\overline{S}\right) - \log\nu(S'_t)}{(1+\rho)^t} \\ > \;& \sum_{t=0}^{t_3-1} \frac{\log(C_{Wt}) - \log(C'_{Wt})}{(1+\rho)^t} + \frac{\log M}{(1+\rho)^{t_3-1}} \frac{1}{\rho} + \sum_{t=t_3}^{\infty} \frac{\log\nu\left(\overline{S}\right) - \log\nu(S'_t)}{(1+\rho)^t} \end{aligned}$$

the first term is bounded, the second one tends to infinity as $\rho \to 0$ and the third term is positive, so for ρ sufficiently small, $U - U'$ is positive.

Similarly, when $\eta < 1$,

$$\begin{aligned} & U - U' \\ = \;& \sum_{t=0}^{t_3-1} \frac{\left(\nu(S_t) C_{Wt}^{1-\eta}\right)^{1-\eta} - \left(\nu(S'_t) C'_{Wt}\right)^{1-\eta}}{(1-\eta)(1+\rho)^t} + \sum_{t=t_3}^{\infty} \frac{\left(\nu\left(\overline{S}\right) C_{Wt}^{1-\eta}\right)^{1-\eta} - \left(\nu(S'_t) C'_{Wt}\right)^{1-\eta}}{(1+\rho)^t (1-\eta)} \\ > \;& \sum_{t=0}^{t_3-1} \frac{\left(\nu(S_t) C_{Wt}^{1-\eta}\right)^{1-\eta} - \left(\nu(S'_t) C'_{Wt}\right)^{1-\eta}}{(1-\eta)(1+\rho)^t} + \frac{(A-\varepsilon)^{1-\eta} \nu\left(\overline{S}\right)^{1-\eta} (1+\rho)}{(1-\eta)\left(1+\rho-(1+g)^{(1-\eta)}\right)} \left(1 - \frac{1}{M^{1-\eta}}\right) \left(\frac{(1+g)^{(1-\eta)}}{1+\rho}\right)^{t_3} \end{aligned}$$

where the first term is bounded, the second tends to infinity when $\rho \to (1+g)^{(1-\eta)} - 1$, therefore for ρ sufficiently small $U - U'$ is positive.

9.7.2 Long-run growth rate in the switch and full specialization case

Consider the case where after a finite number of periods, country X fully specializes in sector G with more innovation in clean than in dirty (so that asymptotically all innovation occurs in clean), while country $(-X)$ specializes and innovates in H only. If this is the case first note that after another finite number of periods, I get $S_t = \overline{S}$, (see Appendix D.2: dirty input production goes to 0), second that A_{XGt} grows like A_{Xct} at a rate $(1+\kappa)^{1-\gamma}-1$ and that $A_{(-X)Ht}$ grows at a rate $(1+\kappa)^{1-\gamma}-1$. World consumption then follows:

$$\begin{aligned} C_{Wt} &= \left(\nu\left(Y_{NGt}+Y_{SGt}\right)^{\frac{\sigma-1}{\sigma}}+(1-\nu)\left(Y_{NHt}+Y_{SHt}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ &= \zeta\left(\nu\left(A_{XGt}K_X^{\alpha}L_X^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}}+(1-\nu)\left(A_{(-X)Ht}K_{(-X)}^{\beta}L_{(-X)}^{1-\beta}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \end{aligned}$$

which grows at a rate $(1+\kappa)^{1-\gamma}-1$.

9.7.3 Allocation in a non full specialization case

Consider any allocation (C'_{Wt}, S'_t). First note that $(1+\kappa)^{1-\gamma}-1$ is the largest possible growth rate for C_{Wt}, since asymptotically $Y_{NGt}+Y_{SGt}$ cannot grow faster than $\max\left(A_{NGt}, A_{SGt}\right)$ and $Y_{NHt}+Y_{SHt}$ cannot grow faster than $\max\left(A_{NHt}, A_{SHt}\right)$, and the fastest asymptotic growth rate for A_{XGt}, A_{XHt} is $(1+\kappa)^{1-\gamma}-1$. Now if either Y_{WGt} or Y_{WHt} does not grow at $(1+\kappa)^{1-\gamma}-1$ asymptotically, then an allocation featuring full specialization and full switch in finite time will necessarily be preferred by the social planner in view of lemma 5 for sufficiently small discount rate ρ.

Now, to get Y_{WGt} growing at a rate $(1+\kappa)^{1-\gamma}-1$, it is necessary that in one country production of good G grows at a rate $(1+\kappa)^{1-\gamma}-1$. Without loss of generality, assume that Y_{NGt} grows at $(1+\kappa)^{1-\gamma}-1$. As Y_{Ndt} must remain bounded to avoid a disaster, Y_{Nct} must grow at a rate $(1+\kappa)^{1-\gamma}$, which requires that A_{Nct} grows at a rate $(1+\kappa)^{1-\gamma}-1$. Similarly to get $Y_{NHt}+Y_{SHt}$ growing at $(1+\kappa)^{1-\gamma}-1$, it is necessary that either A_{NHt} or A_{SHt} also grow at the rate $(1+\kappa)^{1-\gamma}-1$. Since asymptotically all scientists must be allocated to the clean subsector in the North, A_{NHt} cannot grow exponentially and A_{SHt} must be growing at $(1+\kappa)^{1-\gamma}-1$ (and in return, A_{SGt} does grow exponentially). (57) and (58) must then necessarily be satisfied in finite time (since $\frac{\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{1-\delta_{Nt}}$ asymptotically behaves like A_{Nct}), and full specialization is reached.

Therefore the optimal solution for sufficiently small ρ must feature full specialization in finite time with a switch to clean innovation in the country specializing in sector G. Moreover, once the clean technology is sufficiently advanced in the country specialized in sector G, emissions will be negligible so that the quality of the environment will asymptotically go back to $\overline{S}$.

9.8 Appendix B.8: proof of proposition 4

This proof has two steps, first I specify the equilibrium constraints for the South, second I derive the social optimum for the case of the maximization of (1) - the case of maximizing (2) is treated in Appendix D.5

9.8.1 Step 1: Laissez-faire constraints in the South

First I recall the explicit equations for the constraints (29) and (30). Y_{SGt} and Y_{SHt} are given by (65) and (66) if $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \in \left(\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}, \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\beta}(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}\right)$,

$$Y_{SGt} = 0 \text{ and } Y_{SHt} = \zeta A_{SHt} K_S^{\beta} L_S^{1-\beta}$$

if $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \leq \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$ and

$$Y_{SGt} = \zeta A_{SGt} K_S^{\alpha} L_S^{1-\alpha} \tag{78}$$

if $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \geq \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\beta}(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$. This overall delivers the constraint (29) with the function y_{SG} increasing in p_t (weakly), and A_{SGt} and decreasing in A_{SHt} (weakly), and the function y_{SH} decreasing in p_t (weakly) and A_{SGt} (weakly) but increasing in A_{SHt}. y_{SG} and y_{SH} are only piecewise smooth (at the corner of full specialization, the functions are not differentiable). Note that since the South economy maximizes GDP:

$$p_t \frac{\partial y_{SG}}{\partial p} + \frac{\partial y_{SH}}{\partial p} = 0. \tag{79}$$

When $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} > \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\beta}(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$, the allocation of scientists is trivially given by $s_{dt} = 1$ and when $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \leq \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$, by $s_{dt} = 0$. When $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} \in \left(\left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}, \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\beta}(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}\right)$, the allocation of scientists is given by (67), that is:

$$\frac{1+\widetilde{\kappa}(1-s_{SGt})}{\widetilde{\kappa}'(1-s_{SGt})} \frac{\widetilde{\kappa}'\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)}{\left(1+\widetilde{\kappa}\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)\right)} \left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon-1} \tag{80}$$

$$= \frac{\alpha\left(\frac{A_{SHt}^{\alpha}}{A_{SGt}^{\beta}}\right)^{\frac{1}{\alpha-\beta}} L_S - (1-\alpha)\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} \left(\frac{A_{SGt}^{1-\beta}}{A_{SHt}^{1-\alpha}}\right)^{\frac{1}{\alpha-\beta}} p_t^{\frac{1}{\alpha-\beta}} K_S}{(1-\beta)\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}} p_t^{\frac{1}{\alpha-\beta}} \left(\frac{A_{SGt}^{1-\beta}}{A_{SHt}^{1-\alpha}}\right)^{\frac{1}{\alpha-\beta}} K_S - \beta\left(\frac{A_{SHt}^{\alpha}}{A_{SGt}^{\beta}}\right)^{\frac{1}{\alpha-\beta}} L_S}.$$

where $\widetilde{s_{Sdt}}$ is itself define through:

$$\frac{\widetilde{\kappa}'\left(s_{SGt} - \widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)}{1+\widetilde{\kappa}\left(s_{SGt} - \widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)} = \frac{\widetilde{\kappa}'\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)}{1+\widetilde{\kappa}\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)} \left(\frac{A_{Sdt}}{A_{Sct}}\right)^{\varepsilon-1}. \tag{81}$$

This corresponds to the constraint (30).Note that I defined $\widetilde{s_{Sdt}}$ as a function of s_{SGt} and $\frac{A_{Sdt}}{A_{Sct}}$ not of s_{SGt} and $\frac{A_{Sd(t-1)}}{A_{Sc(t-1)}}$ as I did in Appendix B.3, this allows to express s_{SGt} as a function of the current productivity levels, which simplifies considerably the expression of the optimal tariff. I use the tilde to ensure that the difference between the two functions is explicit (however, 80 also implicitly define s_{SGt} as a unique function of p_t and the previous period technology levels). Note that the function s_{SG} (weakly) increases in p_t, and (weakly) decrease in A_{SHt}. It is possible to show that the function s_{SGt} is continuously differentiable. Moreover, (80) can be rewritten as:

$$\left(p_t \frac{\partial y_{SG}}{\partial A_{SHt}} + \frac{\partial y_{SH}}{\partial A_{SHt}}\right) \frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{\left(1+\widetilde{\kappa}\left(s_{SHt}\right)\right)} A_{SHt} \tag{82}$$

$$= \frac{\widetilde{\kappa}'\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)}{\left(1+\widetilde{\kappa}\left(\widetilde{s_{Sdt}}\left(s_{SGt}, \frac{A_{Sdt}}{A_{Sct}}\right)\right)\right)} \left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon-1} A_{SGt}\left(p_t \frac{\partial y_{SG}}{\partial A_{SGt}} + \frac{\partial y_{SH}}{\partial A_{SGt}}\right), \tag{83}$$

which stipulates that for given prices, innovation in the South maximizes current GDP $p_t Y_{SGt} + Y_{SHt}$.

9.8.2 Step 2: Deriving the social optimum

To simplify a bit the exposition, I combine (16) and the emission equation for the South $Y_{Sdt} = \left(A_{Sdt}/A_{SGt}\right)^{\varepsilon} Y_{SGt}$ into:

$$S_t = \max\left(\min\left(\left(1+\Delta\right) S_{t-1} - \xi\left(Y_{Ndt} + \left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon} Y_{SGt}\right), \overline{S}\right), 0\right), \tag{84}$$

I then use the following notations for the Lagrange parameters (the corresponding constraints are in parentheses): λ_{Xt} for (3) - both in North and South -; for the North only: λ_{NHt} (4), λ_{NGt} (7), λ_{Nzt} (8),φ_{Nzit} (9), φ_{NHit} (5), η_{NKt} (11) for capital, η_{NLt} (11) for labor, μ_{Nzit} (13), υ_{Nt} (15); ω_t (84), θ_{NGt}, θ_{NHt}, θ_{SGt} and θ_{SHt} -with obvious subscripts- for the equations in (26), χ_t (27), κ_t (28), λ_{SGt} and λ_{SHt} (29), ϕ_t (30), μ_{SHt}, μ_{Sdt} and μ_{Sct} (31), in addition, the social planner faces the constraints: $0 \leq Y_{NGt}$, $0 \leq Y_{NHt}$, with Lagrange parameters: ι_{NGt}, ι_{NHt}.

As specified above, the functions y_{SG} and y_{SH} are not everywhere differentiable, in the following I therefore use generalized Karush Kuhn Tucker conditions: at point of non differentiability the notation $\frac{\partial y_{SG}}{\partial p_t}$, $\frac{\partial y_{SG}}{\partial A_{SGt}}$, $\frac{\partial y_{SG}}{\partial A_{SHt}}$ refers to elements of a vector $\left(\frac{\partial y_{SG}}{\partial p_t}, \frac{\partial y_{SG}}{\partial A_{SGt}}, \frac{\partial y_{SG}}{\partial A_{SHt}}\right)$ belonging to the Clarke generalized gradient of y_{SG}. Therefore it is still the case at these points that $\frac{\partial y_{SG}}{\partial p_t} \geq 0$, $\frac{\partial y_{SG}}{\partial A_{SGt}} > 0$, $\frac{\partial y_{SG}}{\partial A_{SHt}} \leq 0$.

First order conditions with respect to all the "North" variables, and S_t allow us to recover exactly the same equations as in the first best for the North part of the economy (up to replacing θ_{Gt} and θ_{Ht} by θ_{NGt} and and θ_{NHt}). This shows that the economy in the North is similar to the first best case (with a carbon tax, subsidy to the use of intermediates, and research taxes/subsidies that which can be used to decentralize the equilibrium). I am now going to derive that the social planner creates a wedge between relative prices in the North and in the South. To assess the generality of the results, I will not immediately use the functional form. Moreover taking first order condition with respect to C_{St}, I get:

$$\frac{1}{(1+\rho)^t}\frac{\partial u}{\partial C}\left(C_{Nt}+C_{St},S_t\right)=\frac{\nu\left(S_t\right)^{1-\eta}}{(1+\rho)^t}\left(C_{Nt}+C_{St}\right)^{-\eta}=\lambda_{St}=\lambda_{Nt}\equiv\lambda_t.$$

Taking the first order condition with respect to C_{SHt}, I get:

$$\theta_{SHt}+\kappa_t\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}=\lambda_t\frac{\partial C_S}{\partial C_{SHt}}=\lambda_t\left(1-\nu\right)C_{SHt}^{\frac{-1}{\sigma}}\left(\nu C_{SGt}^{\frac{\sigma-1}{\sigma}}+\left(1-\nu\right)C_{SHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}, \quad (85)$$

and with respect to C_{SGt}:

$$\theta_{SGt}+\kappa_t\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}=\lambda_t\frac{\partial C_S}{\partial C_{SGt}}=\lambda_t\nu C_{SGt}^{\frac{-1}{\sigma}}\left(\nu C_{SGt}^{\frac{\sigma-1}{\sigma}}+\left(1-\nu\right)C_{SHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}. \quad (86)$$

Therefore combining the two:

$$\frac{\theta_{SHt}+\kappa_t\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}{\theta_{SGt}+\kappa_t\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}=\frac{\frac{\partial C_S}{\partial C_{SHt}}}{\frac{\partial C_S}{\partial C_{SGt}}}=\frac{1}{p_t},$$

so that:

$$\kappa_t=\frac{\frac{\theta_{SGt}}{p_t}-\theta_{SHt}}{\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}. \quad (87)$$

First order conditions with respect to Y_{SHt} and Y_{SGt} give:

$$\lambda_{SGt}=\theta_{SGt}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\text{ and }\lambda_{SHt}=\theta_{SHt}, \quad (88)$$

First order conditions with respect to M_{Ht} and M_{Gt} give:

$$p_t\chi_t=\theta_{NGt}-\theta_{SGt}\text{ and }\chi_t=\theta_{NHt}-\theta_{SHt}, \quad (89)$$

so that

$$\frac{\theta_{SGt}}{p_t}-\theta_{SHt}=\frac{\theta_{NGt}}{p_t}-\theta_{NHt}. \quad (90)$$

Finally the first order condition with respect to p_t gives:

$$M_{Gt}\chi_t=\lambda_{SGt}\frac{\partial y_{SG}}{\partial p_t}+\lambda_{SHt}\frac{\partial y_{SH}}{\partial p_t}+\kappa_t+\phi_t\frac{\partial s_{Sdt}}{\partial p_t} \quad (91)$$

Let us denote by $(1+b_t)$ an add valorem tariff (export subsidy) on good G, using (71) and (72) in the North one gets,

$$\frac{\frac{\partial C_N}{\partial C_{NG}}}{\frac{\partial C_N}{\partial C_{NH}}} = \frac{\nu C_{NHt}^{\frac{1}{\sigma}}}{(1-\nu) C_{NGt}^{\frac{1}{\sigma}}} = \frac{\theta_{NGt}}{\theta_{NHt}} = \frac{\widehat{p}_{NGt}}{\widehat{p}_{Ht}} = p_t (1+b_t) . \tag{92}$$

Now plugging (88), (87), (89) and (90) in (91), I get:

$$\begin{aligned} & M_{Gt} \frac{(\theta_{NGt} - \theta_{SGt})}{p_t} \\ = \quad & \left(\theta_{SGt} - \omega_t \xi \left(\frac{A_{Sdt}}{A_{SGt}} \right)^{\varepsilon} \right) \frac{\partial y_{SG}}{\partial p_t} + \theta_{SHt} \frac{\partial y_{SH}}{\partial p_t} + \frac{\frac{\theta_{SGt}}{p_t} - \theta_{SHt}}{\frac{\partial}{\partial C_{SHt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} - \frac{1}{p_t} \frac{\partial}{\partial C_{SGt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}} + \phi_t \frac{\partial s_{SGt}}{\partial p_t}. \end{aligned} \tag{93}$$

Further, using (79), (71) and (72) for the North - replacing θ_{Gt} by θ_{NGt}-, (86), (85), (87) and (92):

$$\begin{aligned} & \frac{M_{Gt}}{p_t} \left(\frac{\partial C_N}{\partial C_{NG}} - \frac{\partial C_S}{\partial C_{SGt}} \right) \\ = \quad & -\frac{\omega_t \xi}{\lambda_t} \left(\frac{A_{Sdt}}{A_{SGt}} \right)^{\varepsilon} \frac{\partial y_{SG}}{\partial p_t} + \frac{\phi_t}{\lambda_t} \frac{\partial s_{SGt}}{\partial p_t} + \frac{\partial C_N}{\partial C_{NH}} b_t \left(p_t \frac{\partial y_{SG}}{\partial p_t} + \frac{\left(1 - \frac{M_{Gt}}{p_t} \frac{\partial}{\partial C_{SGt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} \right)}{\frac{\partial}{\partial C_{SHt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} - \frac{1}{p_t} \frac{\partial}{\partial C_{SGt}} \frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}} \right). \end{aligned} \tag{94}$$

This expression shows that the social planner imposes a wedge between relative prices in the North and in the South. With homothetic preferences, this wedge is entirely created by two terms: $-\omega_t \xi \left(\frac{A_{Sdt}}{A_{SGt}} \right)^{\varepsilon} \frac{\partial y_{SG}}{\partial p_t}$ (which is going to be the environmental motive) and $\phi_t \frac{\partial s_{SGt}}{\partial p_t}$ (which is going to be the innovation motive), indeed if $\omega_t = 0$ and $\phi_t = 0$, the solution to that equation would be $b_t = 0$ -as long as preferences are homothetic, so that $\frac{\partial C_N}{\partial C_{NG}} = \frac{\partial C_S}{\partial C_{SGt}}$ at equal relative price). There is no terms of trade motives for the tariff since the social planner cares equally about consumption in the North and consumption in the South. From here, I use the specific functional forms to get a more explicit formula for the tariff to get (see Appendix D.4.2):

$$
\begin{aligned}
&\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{1+b_t} b_t p_t \frac{\partial y_S}{\partial p_t} \\
&+\left(\begin{array}{c}
\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\left((1+b_t)\nu^{\sigma}+(1+\sigma b_t)(1-\nu)^{\sigma} p_t^{\sigma-1}\right)}{(1+b_t)\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)} \\
-\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}
\end{array}\right) \frac{\nu^{\sigma} Y_{SHt}}{p_t\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)} \\
&+\left(\begin{array}{c}
\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \\
-\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)} \frac{\nu^{\sigma}(1-b_t(\sigma-1))+(1-\nu)^{\sigma} p_t^{\sigma-1}}{\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)}
\end{array}\right) \frac{(1-\nu)^{\sigma} p_t^{\sigma} Y_{SGt}}{p_t\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)} \\
&+\frac{p_t\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} b_t}{D_t\left(1+b_t\right)} \frac{\partial s_{SGt}}{\partial p_t}\left(\begin{array}{c}
A_{SGt} \frac{\partial y_{SG}}{\partial A_{SGt}} \frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}} \frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} \\
-\frac{\partial y_{SG}}{\partial A_{SHt}} \frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))} A_{SHt}
\end{array}\right) \\
=\; & \frac{p_t}{\lambda_t} \omega_t \xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\left(\frac{\partial y_{SG}}{\partial p_t}+\frac{\partial s_{SGt}}{\partial p_t} \frac{1}{D_t}\left(\begin{array}{c}
A_{SGt} \frac{\partial y_{SG}}{\partial A_{SGt}} \frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}} \frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))}-\frac{\partial y_{SG}}{\partial A_{SHt}} \frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))} A_{SHt} \\
+\varepsilon\left(\frac{A_{Sct}}{A_{SGt}}\right)^{\varepsilon-1}\left(\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} \frac{\partial s_{Sdt}}{\partial s_{SGt}}-\frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))} \frac{\partial s_{Sct}}{\partial s_{SGt}}\right) Y_{SGt}
\end{array}\right)\right) \\
&+\frac{\partial s_{SGt}}{\partial p_t} \frac{p_t}{\lambda_t D_t}\left(\begin{array}{c}
\mu_{SHt+1} \frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))} A_{SHt+1} \\
-\left(\mu_{Sdt+1} \frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} A_{Sdt+1} \frac{\partial s_{Sdt}}{\partial s_{SGt}}+\mu_{Sct+1} \frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))} A_{Sct+1} \frac{\partial s_{Sct}}{\partial s_{SGt}}\right)
\end{array}\right) \\
&-\frac{\partial s_{SGt}}{\partial p_t} \frac{p_t(1-\gamma)(\varepsilon-1)}{\lambda_t D_t}\left(\begin{array}{c}
\frac{\widetilde{\kappa}'(s_{Sdt})}{1+\widetilde{\kappa}(s_{Sdt})} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \\
-\frac{\widetilde{\kappa}'(s_{Sct})}{1+\widetilde{\kappa}(s_{Sct})} \frac{\partial s_{Sct}}{\partial s_{SGt}}
\end{array}\right)\left(\begin{array}{c}
\mu_{Sc(t+1)} A_{Sc(t+1)} \frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right)}{1+\widetilde{\kappa}\left(s_{Sc(t+1)}\right)} \\
-\mu_{Sd(t+1)} A_{Sd(t+1)} \frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right)}{1+\widetilde{\kappa}\left(s_{Sd(t+1)}\right)}
\end{array}\right) \frac{A_{Sct}^{\varepsilon-1}}{A_{Sdt}^{\varepsilon-1}} \frac{\partial s_{Sd(t+1)}}{\partial a_t} .
\end{aligned} \tag{95}
$$

D_t given by

$$
\begin{aligned}
D_t \equiv\; & (1-\gamma)^{-1}+\frac{\partial s_{SGt}}{\partial A_{SHt}} \frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{\left(1+\widetilde{\kappa}\left(s_{SHt}\right)\right)} A_{SHt} \\
& -\frac{\widetilde{\kappa}'\left(s_{Sdt}\right)}{\left(1+\widetilde{\kappa}\left(s_{Sdt}\right)\right)} A_{Sdt} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \frac{\partial s_{SGt}}{\partial A_{Sdt}}-\frac{\widetilde{\kappa}'\left(s_{Sct}\right)}{\left(1+\widetilde{\kappa}\left(s_{Sct}\right)\right)} A_{Sct} \frac{\partial s_{Sct}}{\partial s_{SGt}} \frac{\partial s_{SGt}}{\partial A_{Sct}},
\end{aligned} \tag{96}
$$

is strictly positive. On the left-hand side, the first term has the sign of b_t since $p_t \frac{\partial y_S}{\partial p_t} \geq 0$, the second term has the sign of b_t except possibly for b_t sufficiently small (close to -1) when $\sigma < 1$, and the third term has the sign of b_t except possibly for b_t sufficiently large when $\sigma < 1$. The sum of the two terms into brackets, however, always has the sign of b_t and when b_t gets sufficiently small, the South exports good G (so that $\nu^{\sigma} Y_{SHt} < (1-\nu)^{\sigma} p_t^{\sigma} Y_{SGt}$) and imports good G when b_t gets sufficiently large, for all purposes the sum of the second and third term will have the sign of b_t. The forth term also has the sign of b_t. Therefore b_t will have the sign of the right-hand side (RHS). On the RHS, the first term in bracket is weakly positive since $\frac{\partial y_{SG}}{\partial p_t} \geq 0$, $\frac{\partial s_{SGt}}{\partial p_t} \geq 0$, $\frac{\partial y_{SG}}{\partial A_{SGt}} \geq 0$, $\frac{\partial y_{SG}}{\partial A_{SHt}} \leq 0$, $\left(\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} \frac{\partial s_{Sdt}}{\partial s_{SGt}} - \frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))} \frac{\partial s_{Sct}}{\partial s_{SGt}}\right) \geq 0$ and the denominator is positive. This first term therefore pushes towards a positive tariff (this is the environmental term) - with ω_t and τ_t related by (73)-, note that the first part of this term represents the effect of the tariff at given technology, while the second term represents the environmental benefits from reducing innovation in sector G. The second term has the sign of

$$\begin{aligned}&(1+\widetilde{\kappa}(s_{SHt+1}))^{1-\gamma}\mu_{SHt+1}\frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))}A_{SHt}\\&-\left((1+\widetilde{\kappa}(s_{Sdt+1}))^{1-\gamma}\mu_{Sdt+1}\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))}A_{Sdt}\frac{\partial s_{Sdt}}{\partial s_{SGt}}+(1+\widetilde{\kappa}(s_{Sct+1}))^{1-\gamma}\mu_{Sct+1}\frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))}A_{Sct}\frac{\partial s_{Sct}}{\partial s_{SGt}}\right)\end{aligned}$$

which is the difference between the social value of allocating one scientist in sector H instead of sector G, for all the future periods (that is not including the current one). The last term, reflects how the allocation between clean and dirty innovation in the future period is affected by the current number of scientists allocated to sector G, and has the sign of $\mu_{Sc(t+1)}A_{Sc(t+1)}\frac{\widetilde{\kappa}'(s_{Sc(t+1)})}{1+\widetilde{\kappa}(s_{Sc(t+1)})}-\mu_{Sd(t+1)}A_{Sd(t+1)}\frac{\widetilde{\kappa}'(s_{Sd(t+1)})}{1+\widetilde{\kappa}(s_{Sd(t+1)})}$, this could be positive or negative: on one hand, since the South is not going to switch to clean technologies, developing clean technologies in the South has little value for consumption's sake but on the other hand, dirty technologies polluted. However this term vanishes as $A_{Sc(t-1)}/A_{Sd(t-1)}$ goes to 0.

Finally note that when the South is fully specialized- and not at the threshold of specialization ($\left(p_t\frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}}\notin\left(\left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}}\frac{L_S}{K_S},\left(\frac{\beta^\beta(1-\beta)^{(1-\beta)}}{\alpha^\beta(1-\alpha)^{1-\beta}}\right)^{\frac{1}{\alpha-\beta}}\frac{L_S}{K_S}\right)$), $\frac{\partial y_{SG}}{\partial p_t}=0$ and $\frac{\partial s_{SGt}}{\partial p_t}=0$, so that the optimal tariff turns out to be $b_t=0$.

9.9 Appendix B.9: proof of proposition 5

I first study the case $\eta\leq 1$ and then the case $\sigma<1$ and $\eta>1$.

9.9.1 Case $\eta\leq 1$.

The proof follows closely Appendix B.7.

First note that when $\eta=1$, a disaster leads to a utility flow of $-\infty$, so whenever it is possible the social planner will avoid it. When $\eta<1$, a disaster leads to a permanent utility flow of 0, but as intervention in the South is prohibited, it may not be possible to avoid it without affecting previous period utilities. However, one can follow part 1 in that case - which is still true-, and as for sufficiently low discount rate positive growth of the utility flow is preferred, a disaster is avoided. In Part 2, I must specify that it is the North that specializes in G, such an allocation is feasible as explained in proposition 1 iv)), and since the South cannot impose a tax or switch to clean technologies, the case where the specialization would be reversed could not feature a sector G growing at $(1+\kappa)^{1-\gamma}-1$ and avoid an environmental disaster. Part 3 must be slightly amended. I still have that there must be a switch towards clean in the North, but the argumentation for full specialization is a bit more complicated.

As before (57) and (58) will necessarily be satisfied in finite time, so that absent any trade tax, there would be full specialization. A trade tax that leaves the South specialized is never optimal: this is direct from the expression for the optimal tariff when the relative price of good G is below the price that just leads to specialization, in the corner case, the trade tax does not affect production in the South, cannot affect pollution if clean technologies in the North are sufficiently advanced, and is therefore just distorting for world consumption (since the carbon tax in the North becomes negligible, free trade maximizes world consumption and technological level in the North can be maintained at the same level with appropriate policies). Now let us assume that there is a non null trade tax implemented at a time t (sufficiently large) that does lead to some production of good G in the South (although asymptotically resources devoted to that sector must go to zero), and let us consider the alternative case where everything research subsidies in the North are adjusted so that technological levels remain the same and free trade is implemented in every following periods. At given technological levels, a non null trade tax leads to lower world consumption than free trade. Now, when removing the trade tax, technological levels in the South are affected: A_{SGt} decreases and A_{SHt} increases, but in free-trade this leads to even larger level of world consumption. Therefore welfare is higher under that alternative set-up: no trade tax is implemented and full specialization is reached in finite time. All research is allocated to sector H in the South in finite time, and asymptotically, all research is allocated to clean intermediates in the North.

9.9.2 Case $\eta > 1$ and $\sigma < 1$

First note that in this case a disaster leads to a utility flow of $-\infty$, so whenever it is possible the social planner will avoid it.

The proof is done in 5 steps: first I show that in the optimum C_{Wt} must grow without bound, second I show that in the South, Y_{SGt} is bounded and that asymptotically all scientists must be allocated to sector H, third I show that A_{Nct} must grow without bound and that there is a full switch towards clean innovation in the North, forth I show that I must then have exponential growth in A_{Nct} in the North, fifth I conclude that full specialization must be reached in finite time.

Part 1: the optimal solution must feature positive growth I want to show the following lemma:

Lemma 6 *Assume $\eta > 1$ and $\sigma < 1$. Let us consider an allocation (C_{Wt}, S_t) such that there is a t_1, such that for $t > t_1$, $S_t = \overline{S}$ and $\lim C_{Wt} = \infty$. Then for ρ sufficiently small the social planner would rather choose the path (C_{Wt}, S_t) to any other (C'_{Wt}, S'_t) if C'_{Wt} is bounded.*

Proof. Then there is a t_2 and a $M > 0$ such that for all $t > t_2$, $C'_{Wt} < M < \frac{1}{2}C_{Wt}$, $S_t = \overline{S}$. I then get:

$$\begin{aligned} U - U' &= \sum_{t=0}^{t_2-1} \frac{1}{(1+\rho)^t} \left(\frac{(\nu(S_t) C_{Wt})^{1-\eta} - (\nu(S'_t) C'_{Wt})^{1-\eta}}{1-\eta} \right) + \sum_{t=t_2}^{\infty} \frac{1}{(1+\rho)^t} \frac{\left(\nu\left(\overline{S}\right) C_{Wt}\right)^{1-\eta} - (\nu(S'_t) C'_{Wt})^{1-\eta}}{1-\eta} \\ &> \sum_{t=0}^{t_2-1} \frac{1}{(1+\rho)^t} \left(\frac{C_{Wt}^{1-\eta} - (C'_{Wt})^{1-\eta}}{1-\eta} + \nu(S_t) - \nu(S'_t) \right) + \frac{\nu\left(\overline{S}\right)^{1-\eta}}{\eta - 1} \left(1 - \frac{1}{2^{\eta-1}}\right) \frac{1}{(1+\rho)^{t_2-1}} \frac{1}{\rho} \end{aligned}$$

where the first term is bounded, the second tends to infinity when $\rho \to 0$, therefore for ρ sufficiently small $U - U'$ is positive. ■

An allocation leading in finite time to full specialization in sector H in the South, and to full specialization in the North in G is feasible and satisfies the assumption of the lemma. Therefore the optimal solution must feature positive long-run growth in both sectors.

Part 2: scientists must all be allocated to sector H in the South asymptotically This is a direct consequence of lemma 3: A_{SHt} must grow at the rate $(1+\kappa)^{1-\gamma} - 1$ (technically, A_{SHt} may be dominated by $(1+\kappa)^{(1-\gamma)t}$, but grows faster than $M\left((1+\kappa)^{1-\gamma} - x\right)^t$, no matter how small x is).

Part 3: A_{Nct} must grow and switch to clean in the North Note that

$$C_{Wt} = C_{Nt} + C_{St} < 2\left(\nu\left(Y_{NGt} + Y_{SGt}\right)^{\frac{\sigma-1}{\sigma}} + (1-\nu)\left(Y_{NHt} + Y_{SHt}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

so that with $\sigma < 1$, to get C_{Wt} unbounded it is necessary that $Y_{NGt} + Y_{SGt}$ is unbounded, since Y_{SGt} is bounded, I must have Y_{NGt} unbounded. Y_{Ndt} must remain bounded however, so that using (44), $A_{Nct}/\left((1+\tau_{Nt})^{-1} A_{Ndt}\right)$ must tend towards infinity. Using (25), innovation in sector G in the North must switch to being mostly in clean technologies, so that A_{Nct} grows unboundedly, while A_{Ndt} grows infinitely less, and emissions in the North become negligible. A_{Ndt}/A_{Nct} becomes negligible and s_{Ndt} is negligible relative to s_{Nct}.

Part 4: there must be exponential growth in A_{Nct} Assume that this is not the case, then in the North all scientists must be allocated to sector H asymptotically and A_{NHt} must grow faster than $\left((1+\kappa)^{1-\gamma} - x\right)^t$, no matter how small x is. If the North fully specializes in sector G this is clearly not optimal. For the North not to fully specialize while the South does so in H, it is necessary to prevent (57) from holding with $X = N$, since when the South is fully specialized (not at the corner), $b_t = 0$. Note that this is impossible if A_{SHt} and A_{NHt} both grow faster than $\left((1+\kappa)^{1-\gamma} - x\right)^t$ for all $x > 0$, and A_{Nct} does not grow exponentially, in fact if A_{Nct} does not grow exponentially, this case will never be possible after a finite number of periods. Therefore, after a finite number of periods the South does not specialize.

Lemma 3 already demonstrates that the North cannot import the polluting good, therefore the only possibility left is that the South does not specialize and the North alternates periods with non-specialization or full-specialization in G, and $M_{Gt} < 0$. To ensure that A_{Nct} does not grow exponentially, it is necessary that in periods where the North does not specialize, $\liminf s_{Nct} = 0$.

Now I show that if $\frac{p_{NGt}Y_{NGt}}{p_{Ht}Y_{NHt}}$ is bounded below, $\liminf s_{Nct} \neq 0$. Indeed, in the North innovation switches to clean in sector G in finite time, so that in periods where the North does not specialize:

$$\frac{\widetilde{\kappa}'(s_{NHt})}{(1+\widetilde{\kappa}(s_{NHt}))}\frac{1+\widetilde{\kappa}(s_{NGt})}{\widetilde{\kappa}'(s_{NGt})} = \frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}}$$

and to ensure that $\liminf s_{Nct} = 0$, I must have $\liminf \frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}} = 0$. Now:

$$\frac{\lambda_{Nct}Y_{Nct}}{\lambda_{NHt}Y_{NHt}} = \frac{p_{Nct}Y_{Nct}}{p_{NHt}Y_{NHt}} = \frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}\frac{p_{Nct}Y_{Nct}}{p_{NGt}Y_{NGt}},$$

where since $A_{Nct}/\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)$ tends towards infinity $\frac{p_{Nct}Y_{Nct}}{p_{NGt}Y_{NGt}}$ tends toward 1. Now if $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is bounded below, then necessarily $\frac{\lambda_{Nct}Y_{Nct}}{\lambda_{NHt}Y_{NHt}}$ is also bounded below. Denote such a bound by m, using (76), I can write that at a time t where the North does not specialize:

$$\frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}} = \frac{\sum\limits_{t'=t}^{\infty}\lambda_{Nct}Y_{Nct}}{\sum\limits_{t'=t}^{\infty}\lambda_{NHt}Y_{NHt}},$$

since $\mu_{NH0}A_{NH0}$ is finite, $\mu_{NHt}A_{NHt}$ must converge, so that there is a t_2 large enough so that for the t that I am looking at $\mu_{NHt_2}A_{NHt_2} = \sum\limits_{t=t_2}^{\infty}\lambda_{NHt}Y_{NHt} \leq \lambda_{NHt}Y_{NHt} \leq \sum\limits_{t'=t}^{t=t_2-1}\lambda_{NHt}Y_{NHt}$, and

$$\frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}} = \frac{\sum\limits_{t'=t}^{\infty}\lambda_{Nct}Y_{Nct}}{\sum\limits_{t'=t}^{\infty}\lambda_{NHt}Y_{NHt}} > \frac{\sum\limits_{t'=t}^{t_2}\lambda_{Nct}Y_{Nct}}{2\sum\limits_{t'=t}^{t_2}\lambda_{NHt}Y_{NHt}} > \frac{\sum\limits_{t'=t}^{t_2}m\lambda_{NHt}Y_{NHt}}{2\sum\limits_{t'=t}^{t_2}\lambda_{NHt}Y_{NHt}} = \frac{m}{2},$$

so that $\frac{\mu_{Nct}A_{Nct}}{\mu_{NHt}A_{NHt}}$ must be bounded below, and $\liminf s_{Nct} \neq 0$.

Now I am going to show that indeed, when $M_{Gt} \leq 0$, $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ must be bounded below.

First, I can write, using (48) and (49),

$$\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}} = \left(\frac{A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}}{\left(A_{Nct}^{\varepsilon-1}+(1+\tau_{Nt})^{-\varepsilon}A_{Ndt}^{\varepsilon-1}\right)}\right)^{\frac{1}{\varepsilon-1}}$$

$$\times\frac{\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_t(1+b_t)\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{A_{NHt}}\right)^{\frac{1}{\alpha-\beta}}K_N-\beta L_N}{\alpha L_N-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_t(1+b_t)\left(A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}}{A_{NHt}}\right)^{\frac{1}{\alpha-\beta}}K_N}.$$

Now assume that $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is not bounded below, then no matter how small m is, there must be a t such that $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}} < m$. Since $\lim \frac{A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}}{\left(A_{Nct}^{\varepsilon-1}+(1+\tau_{Nt})^{-\varepsilon}A_{Ndt}^{\varepsilon-1}\right)} = 1$, I must get that the numerator must be smaller than some constant times m. This implies that $\left(\frac{p_{NGt}}{p_{NHt}}\frac{A_{NGt}(\tau_{Nt})}{A_{NHt}}\right)^{\frac{1-\alpha}{\alpha-\beta}}$ is bounded above and using (48) and (49) that Y_{NHt}/Y_{NGt} is greater than some constant times the inverse of m. Now, I can also write (for $M_{Gt} \leq 0$ which I have assumed),

$$\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}} = \frac{\nu}{1-\nu}\left(\frac{Y_{NGt}+M_{Gt}}{Y_{NHt}+M_{Ht}}\right)^{-\frac{1}{\sigma}}\frac{Y_{NGt}}{Y_{NHt}} \geq \frac{\nu}{1-\nu}\left(\frac{Y_{NHt}}{Y_{NGt}}\right)^{\frac{1-\sigma}{\sigma}},$$

which must then be greater than some constant times the inverse of $m^{\frac{1-\sigma}{\sigma}}$, clearly that contradicts the assumption that $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is not bounded below.

Therefore $\frac{p_{NGt}Y_{NGt}}{p_{NHt}Y_{NHt}}$ is bounded below, I get a contradiction in all cases and A_{Nct} must grow exponentially.

Part 5 full specialization must be reached in finite time As A_{Nct} grows exponentially (a positive mass of scientists are asymptotically allocated to clean intermediates in the North), then it must be the case that A_{NHt}/A_{SHt} tends towards zero. Therefore, $\frac{\widetilde{K_S}}{\widetilde{L_S}}\frac{\widetilde{L_N}}{K_N}$ must tend towards 0, and (since $\delta_S = 0$ and $\delta_N \to 0$), (53) -with $X = S$- cannot be satisfied, so that asymptotically, one country at least has to specialize and the North has the comparative advantage in producing good G. (57) must be satisfied in finite time (with $X = N$, as $\tau_S = 0$ and $A_{Nct}/A_{Ndt} \to 0$ and $\delta_{Nt} \to 0$), if at some point condition (58) becomes satisfied, than without a trade tax the economy would feature full specialization, and I can apply the same logic as in case $\eta \leq 1$, to conclude that a non null tariff would necessarily be welfare reducing.

Let us then assume that (58) is never satisfied (which requires that A_{Nct} does not grow faster than $A_{NHt}^{1-\sigma}$). This corresponds to a case where without the trade tax, the North would fully specialize in G, while the South would produce both goods. Now if indeed the North were to permanently specialize in G, all innovation in the North would be directed to clean innovation and (58) will end up being satisfied. To ensure that (58) is not satisfied, a trade tax must be imposed in an infinite number of periods. A trade tax is not optimal when the South is specialized and not at the corner of specialization, so that $p_t \frac{A_{SGt}}{A_{SHt}}$ must be bounded below, and as shown in part 2, above too. Moreover, since A_{SGt} does not grow exponentially, p_t must grow at the same rate as A_{SHt}. Knowing that A_{NGt} grows exponentially, to avoid that $p_t (1 + b_t) \frac{A_{NGt}}{A_{NHt}}$ becomes unbounded and that the North specializes in sector G at one point, the trade tax b_t must be negative. I now show that such a tariff cannot be optimal. First note that a negative tariff increases p_t relative to the free trade case, and therefore cannot decrease pollution in the current period (if t is sufficiently large that the North's emissions are negligible). Moreover, world consumption with a trade tax is necessarily smaller than without at given technological levels, that is:

$$C_{Wt|b\neq 0} < \left(\nu \left(\zeta A_{NGt} K_N^{\alpha} L_N^{1-\alpha} + \zeta A_{SGt} K_{SGt}^{\alpha} L_{SGt}^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} + (1-\nu) \left(\zeta A_{SHt} K_{SHt}^{\beta} L_{SHt}^{1-\beta} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

Now consider the alternative allocation where there is free trade and all research in the North is allocated to clean intermediates. Note that a higher A_{NGt} (and lower A_{NHt}) reduces p_t further (and p_t is already reduced by the removal of a negative b_t), so that environmental quality is reduced (since the North does not pollute any more), A_{SHt} increases and A_{SGt} decreases. Now since A_{NGt} grows exponentially while A_{SGt} does not, the increase in A_{NGt} more then compensate for the decrease in A_{SGt}. Since A_{SGt} is lower, p_t is lower and A_{SHt} is larger emissions in the South are always lower in all subsequent periods. Therefore world consumption is larger every period under the alternative path and the quality of the environment weakly higher, as a consequence this case with a negative b_t is never optimal and (58) must be satisfied in finite time.

The economy must then move towards full specialization, so that the tariff becomes useless, environmental quality recovers as emissions are negligible in both countries and the tax becomes null.

Supplementary Appendix

10 Appendix C: Calibration exercise

10.1 Appendix C.1: calibrating the model

The model applies to tradeable goods, so the calibration focuses on emissions from the manufacturing sector. The IEA data provide disaggregated sectoral emissions of CO2 from fossil fuel combustion in the world. Emissions from manufacturing for energy generation are disaggregated in 10 sectors, I base my calibration on these data. Unfortunately, emissions from electricity generation, which are partly eventually used in manufacturing, are counted as a separate sector, so that I cannot integrate them in the analysis. I restrict attention to countries where there is enough disaggregation in manufacturing (so that less of half of the emissions in manufacturing and construction are in the "non specified industry" category within manufacturing and industry), I then scale up emissions in a specified category so as to match the overall level for emissions in manufacturing and construction in the country (for the biggest contributors the share of non specified industry emission is very small). Similarly, I use the UNIDO data to get value-added in each of the sectors defined in the IEA dataset, I get rid of countries for which too many data are missing, and of sectors 25, 33 and 37, which do not correspond to any sector in the IEA dataset, and I fill in with the mean across years for countries where information is missing in a given year. I convert the value added in constant 2000$ dollars. The final list of countries is the following. North (Annex I countries): Australia, Austria, Belgium, Bulgaria, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Latvia, Lithuania, Netherlands, Norway, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States. South (non-Annex I countries): Albania, Azerbaijan, Brazil, China, Colombia, Cyprus, Georgia, India, Indonesia, Macedonia, Mexico, Moldova, Pakistan, Philippines, Qatar, South Africa, South Korea and Thailand.

Aggregating the 51 countries, cumulated emissions (in MtCO2), cumulated value added (in billion of constant 2000$), and emission rates (in MtCO2 per billion dollars) for the years 2003-2007 across the 10 sectors in the IEA data are given by:

Sector	CO2 emissions (Mt)	Value Added (billions of $)	Emission rate
Transport Equipment	300	3544	0.08
Machinery	790	8361	0.09
Wood and Wood Products	161	582	0.28
Food and Tobacco	1518	4150	0.37
Textile and Leather	545	1399	0.39
Paper, Pulp and Printing	1189	2055	0.58
Chemical and Petrochemical	3503	3751	0.93
Non-Ferrous Metals	651	575	1.13
Non-Metallic Minerals	4754	1292	3.68
Iron and Steel	7171	1334	5.37

I identify sector G with Chemical and petrochemical, non-ferrous metals, non-metallic minerals and iron and steel, and the other sectors with sector H (so that I ignore their emissions). Aggregating emissions and value added in sector G for the North and the South, give the emission rates 1.31 and 5.07 respectively.

Aggregating at the North / South level, the ratio of production of nonfossil origin over fossil fuel energy in country primary energy supply are given by 0.27 (in the North) and 0.214 (in the South). I identify the initial A_{Nc0}/A_{Nd0} and A_{Sc0}/A_{Sd0} from these data. The ratio ξ_S/ξ_N is then adjusted so that the ratio of emission rates in the South over the North is equal to 5.07/1.31 as in the data. Finally the levels ξ_S and ξ_N are proportionally adjusted upwards so that the total amount of emissions generated in the model is equal to the total amount of emission generated in reality (including emissions from other sectors, and countries not included in the calibration).

The modelization of environmental costs follows AABH. I consider a disaster temperature of $Temp_{disaster}$ =6°C and relate temperature increase since preindustrial times $Temp$ (in °C) with atmospheric concentration in CO_2 (in ppm) as $Temp \simeq 3\log_2(C_{CO2}/280)$. The quality of the environment is then define as $S = C_{CO2,disaster} - \max\{C_{CO_2}, 280\}$, where $C_{CO2,disaster}$ is the concentration corresponding to the disaster temperature, and 280 is the preindustrial concentration in CO_2 in ppm (so that $S = \overline{S}$ corresponds to preindustrial levels of CO_2 concentration and $S = 0$ to a disaster). Finally the function $v(S_t)$ is chosen to be of the form $v(S) \equiv \frac{(Temp_{disaster}-Temp(S))^{\iota}-\iota\Delta^{\iota-1}_{disaster}(Temp_{disaster}-Temp(S))}{(1-\iota)Temp^{\iota}_{disaster}}$ (which satisfies the assumption that $v(0) = 0$ and $v'\left(\overline{S}\right) = 0$), where ι is computed so as to match DICE model's damage function (Nordhaus (2008)) over the range of temperature increases up to 3°C. This gives a value of $\iota = 0.1443$.

Finally, to avoid the scale effect that arises from the implementation of the subsidy to the use of all intermediates (which corrects for the monopoly distortion), I assume that the optimal subsidy $\widetilde{q} = 1-\gamma$ is implemented in all cases in both countries (everything would be exactly

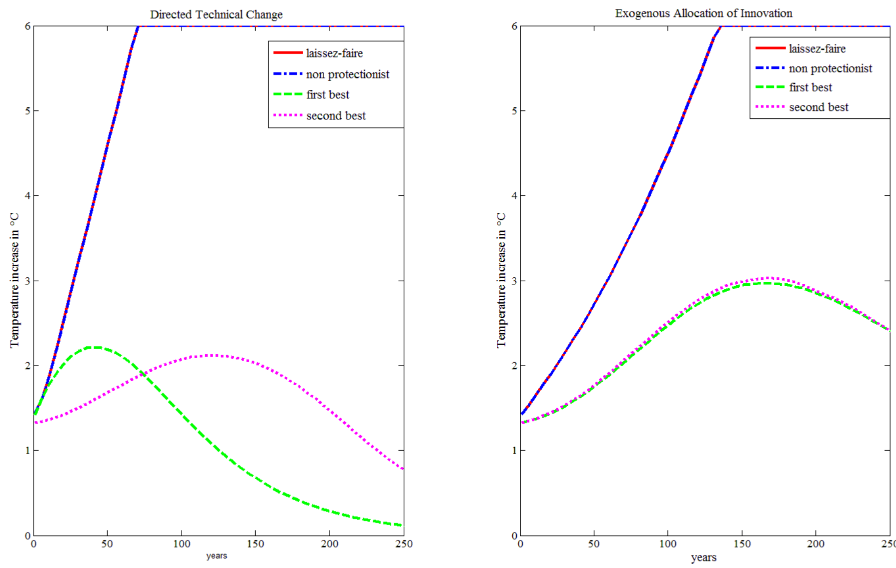

Figure 5: Figure B.1: Increases in Temperature with and without Directed Technical Change

identical as long as the subsidy were to remain the same in a given country in each exercise and in the calibration to the 2003-2007 economy).

10.2 Appendix C.2: DTC as a double edge sword with the original parameters

This subsection carries the same exercise as subsection 5.4 with the calibrated capital shares $\alpha = 0.5$ and $\beta = 0.3$. The only important difference is that in this case, the North cannot slow down a disaster with non-protectionist policies even in the absence of directed technical change.

11 Appendix D: Other proofs and details

11.1 Appendix D.1 Proof of lemma 1

To avoid repetition I prove lemma 1 in a more general context where I allow for a carbon tax and an add-valorem tax on the wage of scientists in dirty research in both countries. Appendix B.1 has already proved that given technological levels the equilibrium is unique. There I show that for κ sufficiently small and $\iota \geq 1/2$, the allocation of scientists is also unique given previous technology levels. This ensures that the entire equilibrium is unique. First, using (63), I get the following relationship between s_{Xdt} and s_{Xct}:

$$\widetilde{\kappa}'(s_{Xct})(1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)-1}A_{Xct-1}^{\varepsilon-1}=\widetilde{\kappa}'(s_{Xdt})(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)-1}(1+\tau_{Xt})^{-\varepsilon}A_{Xdt-1}^{\varepsilon-1}.$$

For κ sufficiently small this equation defines s_{Xct} as an increasing function of s_{Xdt} given the previous technology levels. The derivative of s_{Xct} with respect to s_{Xdt} is then given by:

$$\frac{ds_{Xct}}{ds_{Xdt}}=\frac{\widetilde{\kappa}''(s_{Xdt})+((\varepsilon-1)(1-\gamma)-1)\frac{\widetilde{\kappa}'(s_{Xdt})^2}{1+\widetilde{\kappa}(s_{Xdt})}}{\widetilde{\kappa}''(s_{Xct})+((\varepsilon-1)(1-\gamma)-1)\frac{\widetilde{\kappa}'(s_{Xct})^2}{1+\widetilde{\kappa}(s_{Xct})}}\frac{(1+\widetilde{\kappa}(s_{Xdt}))^{(\varepsilon-1)(1-\gamma)-1}(1+\tau_{Xt})^{-\varepsilon}A_{Xdt-1}^{\varepsilon-1}}{(1+\widetilde{\kappa}(s_{Xct}))^{(\varepsilon-1)(1-\gamma)-1}A_{Xct-1}^{\varepsilon-1}},$$

note that as $\kappa\to 0$, $\frac{\partial s_{Xct}}{\partial s_{Xdt}}\sim\frac{\widetilde{\kappa}''(s_{Xdt})}{\widetilde{\kappa}''(s_{Xct})}\frac{(1+\tau_{Xt})^{-\varepsilon}A_{Xdt-1}^{\varepsilon-1}}{A_{Xct-1}^{\varepsilon-1}}\sim\frac{\widetilde{\kappa}''(s_{Xdt})}{\widetilde{\kappa}''(s_{Xct})}\frac{\widetilde{\kappa}'(s_{Xct})}{\widetilde{\kappa}'(s_{Xdt})}=\frac{s_{Xct}}{s_{Xdt}}$. When country X does not fully specialize, (64) gives the allocation of researchers in country X as $\frac{\widetilde{\kappa}'(1-s_{Xdt}-s_{Xct}(s_{Xdt}))}{1+\widetilde{\kappa}(1-s_{Xdt}-s_{Xct}(s_{Xdt}))}p_{Ht}Y_{XHt}=\frac{\widetilde{\kappa}'(s_{Xdt})}{1+\widetilde{\kappa}(s_{Xdt})}\frac{(1+\tau_{Xt})^{-\varepsilon}A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+(1+\tau_{Xt})^{1-\varepsilon}A_{Xdt}^{\varepsilon-1}}p_{Gt}Y_{XGt}$, which can be rewritten as

$$g_X\left(s_{Xdt},s_{(-X)dt}\right)=0, \tag{97}$$

with

$$g_X\left(s_{Xdt},s_{(-X)dt}\right)\equiv\frac{\widetilde{\kappa}'(1-s_{Xdt}-s_{Xct}(s_{Xdt}))}{1+\widetilde{\kappa}(1-s_{Xdt}-s_{Xct}(s_{Xdt}))}-\frac{\widetilde{\kappa}'(s_{Xdt})f_X\left(A_{XHt},A_{Xdt},A_{Xct},A_{(-X)Ht},A_{(-X)dt},A_{(-X)ct}\right)}{1+\widetilde{\kappa}(s_{Xdt})}$$

and

$$f_X\left(A_{XHt},A_{Xdt},A_{(-X)Ht},A_{(-X)dt}\right)\equiv\frac{(1+\tau_{Xt})^{-\varepsilon}A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+(1+\tau_{Xt})^{1-\varepsilon}A_{Xdt}^{\varepsilon-1}}\frac{p_{Gt}Y_{XGt}}{p_{Ht}Y_{XHt}}.$$

One can then compute the partial derivative of g_X with respect to the allocation of scientists to the dirty sector as:

$$\begin{aligned}&\frac{\partial g_X}{\partial s_{Xdt}}\\=\;&-\frac{\widetilde{\kappa}''(s_{XHt})}{1+\widetilde{\kappa}(s_{XHt})}\left(1+\frac{\partial s_{Xct}}{\partial s_{Xdt}}\right)-\frac{\widetilde{\kappa}''(s_{Xdt})}{1+\widetilde{\kappa}(s_{Xdt})}f_X\left(A_{XHt},A_{Xdt},A_{(-X)Ht},A_{(-X)dt}\right)\\&+\frac{\widetilde{\kappa}'(s_{XHt})^2}{(1+\widetilde{\kappa}(s_{XHt}))^2}\left(1+\frac{\partial s_{Xct}}{\partial s_{Xdt}}\right)+\frac{(\widetilde{\kappa}'(s_{Xdt}))^2}{1+\widetilde{\kappa}(s_{Xdt})}f_X\left(A_{XHt},A_{Xdt},A_{(-X)Ht},A_{(-X)dt}\right)\\&-\frac{(1-\gamma)\widetilde{\kappa}'(s_{Xdt})}{1+\widetilde{\kappa}(s_{Xdt})}\left(\begin{array}{c}-\left(1+\frac{\partial s_{Xct}}{\partial s_{Xdt}}\right)\widetilde{\kappa}'(s_{XHt})\frac{\partial f_X}{\partial A_{XHt}}A_{XHt}+\widetilde{\kappa}'(s_{Xdt})\frac{\partial f_X}{\partial A_{Xdt}}A_{Xdt}\\+\widetilde{\kappa}'(s_{Xct})\frac{\partial s_{Xct}}{\partial s_{Xdt}}\frac{\partial f_X}{\partial A_{Xct}}A_{Xct}\end{array}\right).\end{aligned}$$

For κ small enough and all s_{Xzt} significantly different from 0, $\widetilde{\kappa}'(s_{Xdt})\widetilde{\kappa}'(s_{XHt})$, $\widetilde{\kappa}'(s_{XGt})^2$ and $\widetilde{\kappa}'(s_{XHt})^2$ are negligible relative to $\widetilde{\kappa}''(s_{XHt})$ or $\widetilde{\kappa}''(s_{Xdt})$, and $\widetilde{\kappa}'(s_{Xdt})\widetilde{\kappa}'(s_{Xct})\frac{\partial s_{Xct}}{\partial s_{Xdt}}\sim\kappa^2 s_{Xdt}^{\iota-2}s_{Xct}^{\iota}$ is negligible relative to $\widetilde{\kappa}''(s_{Xdt})$. If s_{XHt} close to 0, then $\widetilde{\kappa}'(s_{XHt})\widetilde{\kappa}'(s_{Xdt})$, $\widetilde{\kappa}'(s_{dt})^2$and $\widetilde{\kappa}'(s_{XHt})^2$ remain negligible relative to $\widetilde{\kappa}''(s_{XHt})$, while if s_{Xdt} close to 0, I similarly have $\widetilde{\kappa}'(s_{XHt})\widetilde{\kappa}'(s_{Xdt})$, $\widetilde{\kappa}'(s_{dt})^2$and $\widetilde{\kappa}'(s_{XHt})^2$ negligible relative to $\widetilde{\kappa}''(s_{Xdt})$ Therefore

g_X decreases in s_{Xdt}, so that (97) defines s_{Xdt} uniquely as a function of $s_{(-X)dt}$. I can then compute:

$$\frac{ds_{Xdt}}{ds_{(-X)dt}} = \frac{(1-\gamma)\frac{\widetilde{\kappa}'(s_{Xdt})}{1+\widetilde{\kappa}(s_{Xdt})}\begin{pmatrix} -\widetilde{\kappa}'\left(s_{(-X)Ht}\right)\left(1+\frac{\partial s_{(-X)ct}}{\partial s_{(-X)dt}}\right)\frac{\partial f_X}{\partial A_{(-X)Ht}}A_{(-X)Ht}+\widetilde{\kappa}'\left(s_{(-X)dt}\right)\frac{\partial f_X}{\partial A_{(-X)dt}}A_{(-X)dt} \\ +\widetilde{\kappa}'\left(s_{(-X)ct}\right)\frac{\partial s_{(-X)ct}}{\partial s_{(-X)dt}}\frac{\partial f_X}{\partial A_{(-X)dt}}A_{(-X)ct}\end{pmatrix}}{\begin{pmatrix} -\frac{\widetilde{\kappa}''(s_{XHt})}{1+\widetilde{\kappa}(s_{XHt})}\left(1+\frac{\partial s_{Xct}}{\partial s_{Xdt}}\right)-\frac{\widetilde{\kappa}''(s_{Xdt})}{1+\widetilde{\kappa}(s_{Xdt})}f_X\left(A_{XHt},A_{Xdt},A_{(-X)Ht},A_{(-X)dt}\right) \\ \frac{\widetilde{\kappa}'(s_{XHt})^2}{(1+\widetilde{\kappa}(s_{XHt}))^2}\left(1+\frac{\partial s_{Xct}}{\partial s_{Xdt}}\right)+\frac{\left(\widetilde{\kappa}'(s_{Xdt})\right)^2}{1+\widetilde{\kappa}(s_{Xdt})}f_X\left(A_{XHt},A_{Xdt},A_{(-X)Ht},A_{(-X)dt}\right) \\ +\frac{(1-\gamma)\widetilde{\kappa}'(s_{Xdt})}{1+\widetilde{\kappa}(s_{Xdt})}\left(\left(1+\frac{\partial s_{Xct}}{\partial s_{Xdt}}\right)\widetilde{\kappa}'\left(s_{XHt}\right)\frac{\partial f_X}{\partial A_{XHt}}A_{XHt}-\widetilde{\kappa}'\left(s_{Xdt}\right)\frac{\partial f_X}{\partial A_{Xdt}}A_{Xdt}-\widetilde{\kappa}'\left(s_{Xct}\right)\frac{\partial s_{Xct}}{\partial s_{Xdt}}\frac{\partial f_X}{\partial A_{Xct}}A_{Xct}\right)\end{pmatrix}}$$

Similarly the allocation o f researchers in country $-X$, needs to solve $g_{-X}\left(s_{(-X)dt}\right)=0$,with

$$g_{-X}\left(s_{(-X)dt}\right)\equiv\frac{\widetilde{\kappa}'\left(1-s_{(-X)dt}\right)}{1+\widetilde{\kappa}\left(1-s_{(-X)dt}\right)}-\frac{\widetilde{\kappa}'\left(s_{(-X)dt}\right)}{1+\widetilde{\kappa}\left(s_{(-X)dt}\right)}f_{-X}\left(A_{XHt},A_{Xdt},A_{Xct},A_{(-X)Ht},A_{(-X)dt},A_{(-X)ct}\right),$$

$$f_{-X}\left(A_{XHt},A_{Xdt},A_{(-X)Ht},A_{(-X)dt}\right)\equiv\frac{\left(1-\tau_{(-X)t}\right)^{-\varepsilon}A_{(-X)dt}^{\varepsilon-1}}{A_{(-X)ct}^{\varepsilon-1}+\left(1-\tau_{(-X)t}\right)^{1-\varepsilon}A_{(-X)dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{(-X)Gt}}{p_{Ht}Y_{(-X)Ht}}.$$

Taking the derivative with respect to $s_{(-X)dt}$, one then gets:

$$\begin{aligned}\frac{dg_{-X}}{ds_{(-X)dt}} = & -\frac{\widetilde{\kappa}''\left(s_{(-X)Ht}\right)}{1+\widetilde{\kappa}\left(s_{(-X)Ht}\right)}\left(1+\frac{\partial s_{(-X)ct}}{\partial s_{(-X)dt}}\right)-\frac{\widetilde{\kappa}''\left(s_{(-X)dt}\right)}{1+\widetilde{\kappa}\left(s_{(-X)dt}\right)}f_{-X} \\ & +\frac{\widetilde{\kappa}'\left(s_{(-X)Ht}\right)^2}{\left(1+\widetilde{\kappa}\left(s_{(-X)Ht}\right)\right)^2}\left(1+\frac{\partial s_{(-X)ct}}{\partial s_{(-X)dt}}\right)+\frac{\left(\widetilde{\kappa}'\left(s_{(-X)dt}\right)\right)}{1+\widetilde{\kappa}\left(s_{(-X)dt}\right)}f_{-X} \\ & -\frac{(1-\gamma)\widetilde{\kappa}'\left(s_{(-X)dt}\right)}{1+\widetilde{\kappa}\left(s_{(-X)dt}\right)}\begin{pmatrix}\begin{pmatrix} -\left(1+\frac{\partial s_{Xct}}{\partial s_{Xdt}}\right)\widetilde{\kappa}'\left(s_{XHt}\right)\frac{\partial f_X}{\partial A_{XHt}}A_{XHt} \\ +\widetilde{\kappa}'\left(s_{Xdt}\right)\frac{\partial f_X}{\partial A_{Xdt}}A_{Xdt}+\widetilde{\kappa}'\left(s_{Xct}\right)\frac{\partial s_{Xct}}{\partial s_{Xdt}}\frac{\partial f_X}{\partial A_{Xct}}A_{Xct}\end{pmatrix}\frac{ds_{Xdt}}{ds_{(-X)dt}} \\ \begin{pmatrix} -\left(1+\frac{\partial s_{(-X)ct}}{\partial s_{(-X)dt}}\right)\widetilde{\kappa}'\left(s_{(-X)Ht}\right)\frac{\partial f_{-X}}{\partial A_{(-X)Ht}}A_{(-X)Ht} \\ +\widetilde{\kappa}'\left(s_{(-X)dt}\right)\frac{\partial f_{-X}}{\partial A_{(-X)dt}}A_{(-X)dt}+\widetilde{\kappa}'\left(s_{-Xct}\right)\frac{\partial s_{(-X)ct}}{\partial s_{(-X)dt}}\frac{\partial f_{-X}}{\partial A_{(-X)ct}}A_{(-X)ct}\end{pmatrix}\end{pmatrix},\end{aligned}$$

and here as well all terms are negligible relative to the first two for κ sufficiently low. Therefore g_{-X} is strictly increasing and $s_{(-X)Gt}$, s_{XGt} are uniquely determined.

When one country fully specialize the same logic applies, all its scientists are allocated to the sector of specialization but the same broad logic applies. At the margin of specialization, one could rewrite

$$\begin{aligned}& g_X\left(s_{Xdt},s_{(-X)dt}\right) \\ \equiv & \frac{1+\widetilde{\kappa}\left(s_{Xdt}\right)}{\widetilde{\kappa}'\left(s_{Xdt}\right)}-\frac{1+\widetilde{\kappa}\left(1-s_{Xdt}-s_{Xct}\left(s_{Xdt}\right)\right)}{\widetilde{\kappa}'\left(1-s_{Xdt}-s_{Xct}\left(s_{Xdt}\right)\right)}f_X\left(A_{XHt},A_{Xdt},A_{Xct},A_{(-X)Ht},A_{(-X)dt},A_{(-X)ct}\right)\end{aligned}$$

- and equivalently for g_{-X}- and I have $g_X\left(s_{Xdt}, s_{(-X)dt}\right)=0$ and $g_{-X}\left(s_{(-X)dt}\right)=0$ in all cases. g_X and g_{-X} are then only piecewise differentiable, but with κ sufficiently small, the function $s_{Xdt}\left(s_{Xct}\right)$ is still increasing, $g_X\left(s_{Xdt}, s_{(-X)dt}\right)$ still decreasing in s_{Xdt} and $g_{-X}\left(s_{(-X)dt}\right)$ is decreasing in $s_{(-X)dt}$, so that there is a unique solution.

The analysis above does not rule out the possibility that there could be both an equilibrium where one country fully specialize and one where there is not full specialization, one can check, however that this cannot be the case if $\iota \geq 1/2$. Further, note that in equilibrium, comparative statics on the allocation of scientists with respect to taxes or previous period technology levels can then be derived by simply ignoring the terms in $(1+\widetilde{\kappa}\left(s_{Xzt}\right))$.

11.2 Appendix D.2 Proof of proposition 1

With $\sigma=1$, the allocation of innovation in the polluting good in the country exporting good G obeys (from (64) and (51))

$$\frac{1+\widetilde{\kappa}\left(s_{XHt}\right)}{\widetilde{\kappa}'\left(s_{XHt}\right)}\frac{\widetilde{\kappa}'\left(s_{Xdt}\right)}{1+\widetilde{\kappa}\left(s_{Xdt}\right)}\frac{A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+A_{Xdt}^{\varepsilon-1}}=\frac{p_{Ht}Y_{XHt}}{p_{Gt}Y_{XGt}}\leq\frac{p_{Ht}\left(Y_{NHt}+Y_{SHt}\right)}{p_{Gt}\left(Y_{NGt}+Y_{SGt}\right)}=\frac{1-\nu}{\nu} \tag{98}$$

if there is not full specialization in the country - with $\frac{A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+A_{Xdt}^{\varepsilon-1}} \geq \frac{1}{2}$-, while if there is full specialization at least half of the scientists are allocated to the dirty subsector, as a consequence there is always a positive mass of scientists allocated to the dirty subsector: A_{Xdt} must grow exponentially in at least one country which during some periods must export good G. In autarky emissions in this country would grow unboundedly, therefore when the country exports the polluting good this is also the case.

With $\sigma<1$, the previous equation writes as:

$$\frac{1+\widetilde{\kappa}\left(s_{XHt}\right)}{\widetilde{\kappa}'\left(s_{XHt}\right)}\frac{\widetilde{\kappa}'\left(s_{Xdt}\right)}{1+\widetilde{\kappa}\left(s_{Xdt}\right)}\frac{A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}+A_{Xdt}^{\varepsilon-1}}=\frac{p_{Ht}Y_{XHt}}{p_{Gt}Y_{XGt}}\leq\frac{1-\nu}{\nu}\left(\frac{Y_{NGt}+Y_{SGt}}{Y_{NHt}+Y_{SHt}}\right)^{\frac{1-\sigma}{\sigma}}. \tag{99}$$

Here as well there is a non negligible amount of innovation in sector d unless $\frac{Y_{NGt}+Y_{SGt}}{Y_{NHt}+Y_{SHt}}$ in which case innovation in both sectors favor the H sector, so that this situation will not be permanent. Therefore in that case too, production of the polluting good and emissions grow unboundedly at least in the country of export.

To prove the second part, I need to show that when all research is conducted in clean intermediates in sector G (both in the North and in the South), emissions remain bounded. For production of good G in country X to become unbounded, it is necessary that A_{Xct} becomes unbounded - while by assumption A_{Xdt} - is bounded. I then get that Y_{XGt}/A_{Xct} must remain bounded, but emissions will be given by $E_{Xt}=\xi\left(\frac{A_{Xdt}}{A_{XGt}}\right)^{\varepsilon}Y_{XGt}$, and I have $E_{Xt}\leq\xi\frac{A_{Xdt}^{\varepsilon}}{\left(1+\frac{A_{Xdt}^{\varepsilon-1}}{A_{Xct}^{\varepsilon-1}}\right)^{\frac{\varepsilon}{\varepsilon-1}}}A_{Xct}^{1-\varepsilon}M$, where M is a bound on $\frac{Y_{XGt}}{A_{Xct}}$, therefore E_{Xt} in that case tends towards 0. If Y_{XGt} were to remain bounded, E_{Xt} would also remain bounded. In both cases, E_{Xt} remains bounded so that for a sufficiently large $\overline{S}$, a disaster is avoided.

11.3 Appendix D.3 Avoiding a disaster with clean research subsidies only, case $\sigma < 1$

11.3.1 Case where the North cannot allocate researchers to an inactive sector

This case follows Appendix B.5. Assume that at some period $(t-1)$:

$$\left(\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\sigma} A_{NH(t-1)} K_N^{\beta} L_N^{1-\beta} > (1+m)\left(\frac{1-\nu}{\nu}\right)^{\sigma}\left(\frac{K_S}{L_S}\right)^{(\alpha-\beta)(1-\sigma)} K_S^{\beta} L_S^{1-\beta} A_{SH(t-1)}^{\sigma} A_{SG(t-1)}^{1-\sigma}, \tag{100}$$

$$A_{NH(t-1)}^{1-\sigma} A_{NG(t-1)}^{\sigma}\left(\frac{L_N}{K_N}\right)^{(\alpha-\beta)(1-\sigma)} K_N^{\alpha} L_N^{1-\alpha}(1+m) < \left(\frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\sigma}\left(\frac{1-\nu}{\nu}\right)^{\sigma} A_{SG(t-1)} K_S^{\alpha} L_S^{1-\alpha}, \tag{101}$$

where $m > 0$, is sufficiently large that regardless of the allocation of research in the following period, (57) and (58) are satisfied (with $\tau_S = 0$). Then the North fully specializes in H and the South in G, therefore, all scientists in the South are allocated to sector G. With $\sigma < 1$, then the RHS of (100) grows, but if the North cannot allocate scientists to sector G - which is shut down -, the LHS grows faster and the economy remains fully specialized.

11.3.2 Case $\sigma < 1$ if the North can allocate its research even to a sector with zero production

First I show that it is impossible for the South to keep its comparative advantage in sector G in the long-run when the North allocates all its scientists to the clean subsector. Maintaining such a comparative advantage would require keeping $\frac{\widetilde{K_S}}{\widetilde{L_S}} > \frac{\widetilde{K_N}}{\widetilde{L_N}}$, so that asymptotically all scientists in the South would have to innovate in the dirty subsector, which as demonstrated below is impossible.

If indeed, all innovation in the South asymptotically occurs in dirty technologies (55) (with $X = S$ and $\tau_S = 0$) must be violated, so the South cannot specialize in sector G while the North produces both goods in the long-run. However, if there is no innovation in sector H in the North, (57) (with $X = S$ and $\tau_S = 0$) cannot hold in the long-run either, so that it is impossible for both countries to specialize.

Consider now the case where both countries are not fully specialized, then with innovation occurring in sector G only, $\frac{\widetilde{K_X}}{\widetilde{L_X}} \to \infty$ and the condition (54) translates into $\frac{\widetilde{K_S}}{\widetilde{L_S}} \leq \frac{\widetilde{K_N}+\widetilde{K_S}}{\widetilde{L_N}+\widetilde{L_S}}$. This asymptotic inequality cannot be satisfied unless $\frac{\widetilde{K_S}}{\widetilde{L_S}} \to \frac{\widetilde{K_N}}{\widetilde{L_N}}$ (in periods where there is not full specialization). (52) (with $\tau_S = \tau_N = 0$) gives that the price ratio must satisfy:

$\left(\frac{p_G}{p_H}\right)^{1-\sigma}\left(\frac{\nu}{1-\nu}\right)^{\sigma}=\frac{\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_G}{p_H}\right)^{\frac{1}{\alpha-\beta}}\frac{\widetilde{K_N}+\widetilde{K_S}}{\widetilde{L_N}+\widetilde{L_S}}-\beta}{\alpha-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_G}{p_H}\right)^{\frac{1}{\alpha-\beta}}\frac{\widetilde{K_N}+\widetilde{K_S}}{\widetilde{L_N}+\widetilde{L_S}}}$ which implies that $\frac{p_G}{p_H}\to 0$ (in these periods). In this case $\frac{p_G}{p_H}\frac{Y_{SG}}{Y_{SH}}=\frac{\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_G}{p_H}\right)^{\frac{1}{\alpha-\beta}}\frac{\widetilde{K_S}}{\widetilde{L_S}}-\beta}{\alpha-\left(\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\alpha)\left(\frac{p_G}{p_H}\right)^{\frac{1}{\alpha-\beta}}\frac{\widetilde{K_S}}{\widetilde{L_S}}}$ also tends towards 0, and during these periods where there is not full specialization, it is impossible that all scientists gets allocated to the dirty subsector.

Therefore it must be the case that in nearly all periods, the North fully specializes in sector H, while the South produces both goods. (64) can then be rewritten as:

$$\frac{1+\widetilde{\kappa}\left(s_{SHt}\right)}{\widetilde{\kappa}'\left(s_{SHt}\right)}\frac{\widetilde{\kappa}'\left(s_{SGt}\right)}{1+\widetilde{\kappa}\left(s_{SGt}\right)}\frac{A_{Sdt}^{\varepsilon-1}}{A_{Sct}^{\varepsilon-1}+A_{Sdt}^{\varepsilon-1}}=\frac{1-\nu}{\nu}\left(\frac{p_{Gt}}{p_{Ht}}\right)^{\sigma-1}\frac{Y_{SHt}}{Y_{NHt}+Y_{SHt}}, \tag{102}$$

since research in clean technologies in the South gets arbitrarily low. In order to ensure that nearly all scientists get allocated to the dirty subsector in the South, it is necessary that the RHS gets arbitrarily small. As Y_{NHt} is bounded, and as $\frac{p_{Gt}}{p_{Ht}}$ cannot become unbounded (otherwise following (49), the South would have to be specialized in sector G), the only way this can be achieved is by having Y_{SHt} becoming arbitrarily small. From (49) - and since $\frac{p_{Gt}}{p_{Ht}}A_{SGt}$ cannot become unbounded without leading to full specialization -, this can only be achieved if $\left(\frac{p_{Gt}}{p_{Ht}}\frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}}$ becomes arbitrarily close to $\left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{\frac{1}{\alpha-\beta}}\frac{\alpha L_S}{(1-\alpha)K_S}$. Plugging that in the expression for relative prices when the South specializes in the non-polluting sector leads to:

$$A_{SGt}^{1-\sigma}\to\frac{A_{NHt}A_{SHt}^{-\sigma}L_N^{\beta}K_N^{1-\beta}}{K_S^{\alpha(1-\sigma)+\sigma\beta}L_S^{1-\sigma\beta-\alpha(1-\sigma)}}\left(\frac{\nu}{1-\nu}\right)^{\sigma}\left(\frac{\alpha^{\beta}\left(1-\alpha\right)^{1-\beta}}{\beta^{\beta}\left(1-\beta\right)^{(1-\beta)}}\right)^{\sigma}, \tag{103}$$

which is impossible since A_{SGt} is the only term growing at an exponential rate. As a consequence, there is a contradiction: it is impossible to have nearly all scientists allocated to the dirty subsector in the long-run in the South, and at some point $\frac{\widetilde{K_{Nt}}}{\widetilde{L_{Nt}}}>\frac{\widetilde{K_{St}}}{\widetilde{L_{St}}}$.

Knowing that both $\frac{A_{Nd(t-1)}}{A_{Nc(t-1)}}$ and $\frac{A_{Sc(t-1)}}{A_{Sd(t-1)}}$ are negligible and using lemma 2, the South ends up specializing in the non-polluting sector.

11.4 Appendix D.4 Complements on the second best policy

11.4.1 Appendix D.4.1 Deriving (95)

Here I derive the algebra from (94) to (95).

Note that:

$$\frac{\partial C_N}{\partial C_{NG}} = \nu C_{NGt}^{\frac{-1}{\sigma}}\left(\nu C_{NGt}^{\frac{\sigma-1}{\sigma}}+(1-\nu)\,C_{NHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}, \quad (104)$$

$$\frac{\partial C_N}{\partial C_{NH}} = (1-\nu)\,C_{NHt}^{\frac{-1}{\sigma}}\left(\nu C_{NGt}^{\frac{\sigma-1}{\sigma}}+(1-\nu)\,C_{NHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{p_t\left(1+b_t\right)}. \quad (105)$$

$$\frac{\partial C_S}{\partial C_{SG}} = \nu C_{SGt}^{\frac{-1}{\sigma}}\left(\nu C_{SGt}^{\frac{\sigma-1}{\sigma}}+(1-\nu)\,C_{SHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}, \quad (106)$$

$$\frac{\partial C_S}{\partial C_{SH}} = (1-\nu)\,C_{SHt}^{\frac{-1}{\sigma}}\left(\nu C_{SGt}^{\frac{\sigma-1}{\sigma}}+(1-\nu)\,C_{SHt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{p_t}. \quad (107)$$

therefore

$$\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} = -\frac{1}{\sigma}\frac{1}{C_{SGt}}p_t \text{ and } \frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}} = \frac{1}{\sigma}\frac{1}{C_{SHt}}p_t. \quad (108)$$

Using these equations in (94), I get:

$$\begin{aligned}&\frac{M_{Gt}}{p_t}\left(\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}-\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\right)\\ =\; & -\frac{\omega_t\xi}{\lambda_t}\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}+\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{p_t\left(1+b_t\right)}b_t\left(p_t\frac{\partial y_{SG}}{\partial p_t}+\frac{(1-\nu)^{\sigma}\left(\sigma C_{SGt}+M_{Gt}\right)}{p_t^{1-\sigma}\nu^{\sigma}+(1-\nu)^{\sigma}}\right)\\ &+\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t}.\end{aligned}$$

therefore using (26):

$$\begin{aligned}&\left(\begin{array}{c}\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t(1+b_t)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{1+b_t}\frac{(1+b_t)\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}(1+\sigma b_t)}{\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}}\\ -\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\end{array}\right)\frac{M_{Gt}}{p_t} \quad (109)\\ =\; & b_t\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{p_t\left(1+b_t\right)}\left(p_t\frac{\partial y_{SG}}{\partial p_t}+\frac{(1-\nu)^{\sigma}p_t^{\sigma-1}\sigma Y_{SGt}}{\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}}\right)\\ &+\frac{1}{\lambda_t}\left(\phi_t\frac{\partial s_{SGt}}{\partial p_t}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}\right)\end{aligned}$$

Similarly, plugging (27), (28), (79), (88), (89) and (90) in (91) gives:

$$-M_{Ht}\left(\theta_{NHt}-\theta_{SHt}\right) = p_t\left(\theta_{NHt}-\frac{\theta_{NGt}}{p_t}\right)\frac{\partial y_{SH}}{\partial p}+p_t\kappa_t+p_t\left(\phi_t\frac{\partial s_{SGt}}{\partial p_t}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}\right). \quad (110)$$

Further using (72) for the North with θ_{NHt} instead of θ_{Ht}, and (85), (87), (90) and (92) gives:

$$-M_{Ht}\left(\frac{\partial C_N}{\partial C_{NHt}}-\frac{\partial C_S}{\partial C_{SHt}}\right) \tag{111}$$
$$= b_t\frac{\partial C_N}{\partial C_{NHt}}\left(-p_t\frac{\partial y_{SH}}{\partial p}+\frac{p_t+M_{Ht}\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}{\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}\right)+\frac{p_t}{\lambda_t}\left(\phi_t\frac{\partial s_{SGt}}{\partial p_t}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}\right).$$

Now plugging (105), (106), (107), (108) and (26), I get:

$$\frac{M_{Ht}}{p_t}\left(\begin{array}{c}\frac{\left(\nu^\sigma+(1-\nu)^\sigma(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)}\frac{\nu^\sigma(1+(1-\sigma)b_t)+p_t^{\sigma-1}(1-\nu)^\sigma}{\nu^\sigma+p_t^{\sigma-1}(1-\nu)^\sigma}\\ -\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\end{array}\right) \tag{112}$$
$$= b_t\frac{\left(\nu^\sigma+(1-\nu)^\sigma\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)}\left(\frac{\partial y_{SH}}{\partial p}-\frac{\nu^\sigma\sigma Y_{SHt}}{p_t\left(\nu^\sigma+p_t^{\sigma-1}\left(1-\nu\right)^\sigma\right)}\right)$$
$$-\frac{p_t}{\lambda_t}\left(\phi_t\frac{\partial s_{SGt}}{\partial p_t}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}\right)$$

I combine (27), (109), and (112) to get:

$$\frac{\left(\nu^\sigma+(1-\nu)^\sigma\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{1+b_t}b_tp_t\frac{\partial y_S}{\partial p_t} \tag{113}$$
$$+\left(\begin{array}{c}\frac{\left(\nu^\sigma+(1-\nu)^\sigma(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{1+b_t}\frac{(1+b_t)\nu^\sigma+(1+\sigma b_t)(1-\nu)^\sigma p_t^{\sigma-1}}{\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}}\\ -\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\end{array}\right)\frac{\nu^\sigma Y_{SHt}}{p_t\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)}$$
$$+\left(\begin{array}{c}\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\\ -\frac{\left(\nu^\sigma+(1-\nu)^\sigma(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)}\frac{\nu^\sigma(1-b_t(\sigma-1))+(1-\nu)^\sigma p_t^{\sigma-1}}{\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)}\end{array}\right)\frac{(1-\nu)^\sigma p_t^\sigma Y_{SGt}}{p_t\left(\nu^\sigma+(1-\nu)^\sigma p_t^{\sigma-1}\right)}$$
$$= \frac{p_t}{\lambda_t}\left(\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}-\phi_t\frac{\partial s_{SGt}}{\partial p_t}\right).$$

To establish the sign of the tariff I then still have to express the term $\phi_t\frac{\partial s_{SGt}}{\partial p_t}$. The first order condition with respect to s_{SGt} gives:

$$\phi_t=(1-\gamma)\left(\mu_{Sdt}\frac{\widetilde{\kappa}'\left(s_{Sdt}\right)}{(1+\widetilde{\kappa}\left(s_{Sdt}\right))}A_{Sdt}\frac{\partial s_{Sdt}}{\partial s_{SGt}}+\mu_{Sct}\frac{\widetilde{\kappa}'\left(s_{Sct}\right)}{(1+\widetilde{\kappa}\left(s_{Sct}\right))}A_{Sct}\frac{\partial s_{Sct}}{\partial s_{SGt}}-\mu_{SHt}\frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{(1+\widetilde{\kappa}\left(s_{SHt}\right))}A_{SHt}\right). \tag{114}$$

While the first order condition with respect to A_{SHt} and A_{Sdt} deliver the law of motion for the social value of one unit of average productivity in sector H (μ_{SHt}) and in sector d (μ_{Sdt}) in the South as:

$$\mu_{SHt} = \lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{SHt}} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{SHt}} + \phi_t \frac{\partial s_{SGt}}{\partial A_{SHt}} + \left(1 + \widetilde{\kappa}\left(s_{SHt+1}\right)\right)^{1-\gamma} \mu_{SHt+1}, \tag{115}$$

$$\begin{aligned} \mu_{Sdt} \quad = \quad & \lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{Sdt}} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{Sdt}} + \phi_t \frac{\partial s_{SGt}}{\partial A_{Sdt}} - \omega_t \xi \frac{\varepsilon A_{Sdt}^{\varepsilon-1} A_{Sct}^{\varepsilon-1}}{A_{SGt}^{2\varepsilon-1}} Y_{SGt} + \left(1 + \widetilde{\kappa}\left(s_{Sdt+1}\right)\right)^{1-\gamma} \mu_{Sdt+1} \\ & + \mu_{Sdt+1} (1-\gamma) \frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right)}{\left(1 + \widetilde{\kappa}\left(s_{Sd(t+1)}\right)\right)} A_{Sdt+1} (1-\varepsilon) A_{Sdt}^{-\varepsilon} A_{Sct}^{\varepsilon-1} \frac{\partial s_{Sd(t+1)}}{\partial a_t} \\ & + \mu_{Sc(t+1)} \frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right)}{1 + \widetilde{\kappa}\left(s_{Sc(t+1)}\right)} A_{Sc(t+1)} (1-\varepsilon) A_{Sdt}^{-\varepsilon} A_{Sct}^{\varepsilon-1} (1-\gamma) \frac{\partial s_{Sc(t+1)}}{\partial a_t} \end{aligned} \tag{116}$$

$$\begin{aligned} \mu_{Sct} \quad = \quad & \lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{Sct}} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{Sct}} + \phi_t \frac{\partial s_{SGt}}{\partial A_{Sct}} + \omega_t \xi \frac{\varepsilon A_{Sdt}^{\varepsilon} A_{Sct}^{\varepsilon-2}}{A_{SGt}^{2\varepsilon-1}} Y_{SGt} + \left(1 + \widetilde{\kappa}\left(s_{Sct+1}\right)\right)^{1-\gamma} \mu_{Sct+1} \\ & + (1-\gamma) \mu_{Sd(t+1)} \frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right)}{1 + \widetilde{\kappa}\left(s_{Sd(t+1)}\right)} A_{Sd(t+1)} (\varepsilon - 1) \frac{A_{Sct}^{\varepsilon-2}}{A_{Sdt}^{\varepsilon-1}} \frac{\partial s_{Sd(t+1)}}{\partial a_t} \\ & + \mu_{Sc(t+1)} \frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right)}{1 + \widetilde{\kappa}\left(s_{Sc(t+1)}\right)} (1-\gamma) A_{Sc(t+1)} (\varepsilon - 1) \frac{A_{Sct}^{\varepsilon-2}}{A_{Sdt}^{\varepsilon-1}} \frac{\partial s_{Sc(t+1)}}{\partial a_t} \end{aligned}$$

Combining the last four equations, I get:

$$\begin{aligned} & \phi_t \begin{pmatrix} (1-\gamma)^{-1} + \frac{\partial s_{SGt}}{\partial A_{SHt}} \frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))} A_{SHt} \\ -\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} A_{Sdt} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \frac{\partial s_{SGt}}{\partial A_{Sdt}} - \frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))} A_{Sct} \frac{\partial s_{Sct}}{\partial s_{SGt}} \frac{\partial s_{SGt}}{\partial A_{Sct}} \end{pmatrix} \\ = \quad & \left(\lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{Sdt}} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{Sdt}} - \omega_t \xi \frac{\varepsilon A_{Sdt}^{\varepsilon-1} A_{Sct}^{\varepsilon-1}}{A_{SGt}^{2\varepsilon-1}} Y_{SGt} + \left(1 + \widetilde{\kappa}\left(s_{Sdt+1}\right)\right)^{1-\gamma} \mu_{Sdt+1} \right) \frac{\widetilde{\kappa}'\left(s_{Sdt}\right) A_{Sdt}}{\left(1 + \widetilde{\kappa}\left(s_{Sdt}\right)\right)} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \\ & + \left(\lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{Sct}} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{Sct}} + \omega_t \xi \frac{\varepsilon A_{Sdt}^{\varepsilon} A_{Sct}^{\varepsilon-2}}{A_{SGt}^{2\varepsilon-1}} Y_{SGt} + \left(1 + \widetilde{\kappa}\left(s_{Sct+1}\right)\right)^{1-\gamma} \mu_{Sct+1} \right) \frac{\widetilde{\kappa}'\left(s_{Sct}\right) A_{Sct}}{\left(1 + \widetilde{\kappa}\left(s_{Sct}\right)\right)} \frac{\partial s_{Sct}}{\partial s_{SGt}} \\ & - \left(\lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{SHt}} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{SHt}} + \left(1 + \widetilde{\kappa}\left(s_{SHt+1}\right)\right)^{1-\gamma} \mu_{SHt+1} \right) \frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{\left(1 + \widetilde{\kappa}\left(s_{SHt}\right)\right)} A_{SHt} \\ & + (1-\gamma)(\varepsilon-1) \begin{pmatrix} \frac{\widetilde{\kappa}'(s_{Sct})}{1+\widetilde{\kappa}(s_{Sct})} \frac{\partial s_{Sct}}{\partial s_{SGt}} \\ -\frac{\widetilde{\kappa}'(s_{Sdt})}{1+\widetilde{\kappa}(s_{Sdt})} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \end{pmatrix} \begin{pmatrix} \frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right) \mu_{Sd(t+1)} A_{Sd(t+1)}}{1+\widetilde{\kappa}\left(s_{Sd(t+1)}\right)} \\ -\frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right) \mu_{Sc(t+1)} A_{Sc(t+1)}}{1+\widetilde{\kappa}\left(s_{Sc(t+1)}\right)} \end{pmatrix} \frac{A_{Sct}^{\varepsilon-1}}{A_{Sdt}^{\varepsilon-1}} \frac{\partial s_{Sd(t+1)}}{\partial a_t}. \end{aligned} \tag{117}$$

It is possible to show that when a becomes arbitrarily small $\left(\frac{\widetilde{\kappa}'(s_{Sct})}{1+\widetilde{\kappa}(s_{Sct})} \frac{\partial s_{Sct}}{\partial s_{SGt}} - \frac{\widetilde{\kappa}'(s_{Sdt})}{1+\widetilde{\kappa}(s_{Sdt})} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \right)$ is bounded above and below while $\frac{A_{Sct}^{\varepsilon-1}}{A_{Sdt}^{\varepsilon-1}} \frac{\partial s_{Sd(t+1)}}{\partial a_t} \to 0$. Therefore, the last term tends towards 0. Recall that I have assumed that κ is sufficiently small to ensure that s_{SGt} is uniquely defined given previous technological levels as an increasing function of p_t. This assumption implies that D_t defined in (96) is positive, as s_{SGt} is a function of p_t and the current technological levels. From (117), and using that $\frac{\partial A_{SG}}{\partial A_{Sd}} = \frac{A_{Sdt}^{\varepsilon-2}}{A_{SGt}^{\varepsilon-2}}$, I get:

$$\begin{aligned} & \phi_t D_t \qquad (118) \\ = & \lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{SGt}} \frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}} \frac{\widetilde{\kappa}'(s_{Sdt}) A_{SGt}}{(1+\widetilde{\kappa}(s_{Sdt}))} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{SGt}} \frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}} \frac{\widetilde{\kappa}'(s_{Sdt}) A_{SGt}}{(1+\widetilde{\kappa}(s_{Sdt}))} \\ & + \omega_t \xi \varepsilon \frac{A_{Sdt}^{\varepsilon} A_{Sct}^{\varepsilon-1}}{A_{SGt}^{2\varepsilon-1}} \left(\frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))} \frac{\partial s_{Sct}}{\partial s_{SGt}} - \frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \right) Y_{SGt} \\ & + (1+\widetilde{\kappa}(s_{Sdt+1}))^{1-\gamma} \mu_{Sdt+1} \frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} A_{Sdt} \frac{\partial s_{Sdt}}{\partial s_{SGt}} + (1+\widetilde{\kappa}(s_{Sct+1}))^{1-\gamma} \mu_{Sct+1} \frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))} A_{Sct} \frac{\partial s_{Sct}}{\partial s_{SGt}} \\ & - \left(\lambda_{SGt} \frac{\partial y_{SG}}{\partial A_{SHt}} + \lambda_{SHt} \frac{\partial y_{SH}}{\partial A_{SHt}} + (1+\widetilde{\kappa}(s_{SHt+1}))^{1-\gamma} \mu_{SHt+1} \right) \frac{\widetilde{\kappa}'(s_{SHt})}{(1+\widetilde{\kappa}(s_{SHt}))} A_{SHt} \\ & + (1-\gamma)(\varepsilon-1) \begin{pmatrix} \frac{\widetilde{\kappa}'(s_{Sct})}{1+\widetilde{\kappa}(s_{Sct})} \frac{\partial s_{Sct}}{\partial s_{SGt}} \\ - \frac{\widetilde{\kappa}'(s_{Sdt})}{1+\widetilde{\kappa}(s_{Sdt})} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \end{pmatrix} \begin{pmatrix} \frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right) \mu_{Sd(t+1)} A_{Sd(t+1)}}{1+\widetilde{\kappa}\left(s_{Sd(t+1)}\right)} \\ - \frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right) \mu_{Sc(t+1)} A_{Sc(t+1)}}{1+\widetilde{\kappa}\left(s_{Sc(t+1)}\right)} \end{pmatrix} \frac{A_{Sct}^{\varepsilon-1}}{A_{Sdt}^{\varepsilon-1}} \frac{\partial s_{Sd(t+1)}}{\partial a_t} \end{aligned}$$

Furthermore, combining (87), (90),(92), and (72) for the North -, I can express κ_t as:

$$\kappa_t = \sigma \frac{(1-\nu) C_{SHt}^{\frac{\sigma-1}{\sigma}}}{\left(\nu C_{SGt}^{\frac{\sigma-1}{\sigma}} + (1-\nu) C_{SHt}^{\frac{\sigma-1}{\sigma}} \right)} b_t \lambda_t \frac{\left(\nu^{\sigma} + (1-\nu)^{\sigma} (p_t (1+b_t))^{\sigma-1} \right)^{\frac{1}{\sigma-1}}}{p_t (1+b_t)}$$

which allows us to express θ_{SHt} and θ_{SGt} as (using (28), (86) and (85)

$$\begin{aligned} \theta_{SHt} &= \frac{\lambda_t}{p_t} \left(\left(\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - \frac{\nu^{\sigma}}{\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1}} \frac{b_t}{(1+b_t)} \left(\nu^{\sigma} + (1-\nu)^{\sigma} (p_t (1+b_t))^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \right), \\ & \qquad (119) \\ \theta_{SGt} &= \lambda_t \left(\left(\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1} \right)^{\frac{1}{\sigma-1}} + \frac{(1-\nu)^{\sigma} p_t^{\sigma-1}}{\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1}} \frac{b_t}{(1+b_t)} \left(\nu^{\sigma} + (1-\nu)^{\sigma} (p_t (1+b_t))^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \right). \end{aligned}$$

Combining these two equations with (88) gives:

$$\frac{\lambda_{SGt}}{\lambda_{SHt}} = p_t \begin{pmatrix} 1 + \frac{b_t}{1+b_t} \left(\left(\frac{\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1}}{\nu^{\sigma} + (1-\nu)^{\sigma} (p_t (1+b_t))^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} - \frac{\nu^{\sigma}}{\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1}} \frac{b_t}{(1+b_t)} \right)^{-1} \\ -\omega_t \xi \left(\frac{A_{Sdt}}{A_{SGt}} \right)^{\varepsilon} \lambda_t^{-1} \left(\left(\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - \frac{\nu^{\sigma} \left(\nu^{\sigma} + (1-\nu)^{\sigma} (p_t (1+b_t))^{\sigma-1} \right)^{\frac{1}{\sigma-1}}}{\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1}} \frac{b_t}{(1+b_t)} \right)^{-1} \end{pmatrix}$$

Plugging this equation in (118) and using (82), (88), (119), I get:

$$
\begin{aligned}
& \phi_t D_t \qquad (120) \\
= \; & \frac{\lambda_t \left(\nu^{\sigma} + (1-\nu)^{\sigma} \left(p_t \left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} b_t}{1+b_t} \left(A_{SGt} \frac{\partial y_{SG}}{\partial A_{SGt}} \frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}} \frac{\widetilde{\kappa}'\left(s_{Sdt}\right)}{\left(1+\widetilde{\kappa}\left(s_{Sdt}\right)\right)} - \frac{\partial y_{SG}}{\partial A_{SHt}} \frac{\widetilde{\kappa}'\left(s_{SHt}\right)}{\left(1+\widetilde{\kappa}\left(s_{SHt}\right)\right)} A_{SHt}\right) \\
& -\omega_t \xi \left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon} \left(\begin{array}{c} \frac{\partial y_{SG}}{\partial A_{SGt}} \frac{A_{Sdt}^{\varepsilon-1}}{A_{SGt}^{\varepsilon-1}} \frac{\widetilde{\kappa}'(s_{Sdt}) A_{SGt}}{(1+\widetilde{\kappa}(s_{Sdt}))} - \frac{\partial y_{SG}}{\partial A_{SHt}} \frac{\widetilde{\kappa}'(s_{SHt}) A_{SHt}}{(1+\widetilde{\kappa}(s_{SHt}))} \\ +\varepsilon \left(\frac{A_{Sct}}{A_{SGt}}\right)^{\varepsilon-1} \left(\frac{\widetilde{\kappa}'(s_{Sdt})}{(1+\widetilde{\kappa}(s_{Sdt}))} \frac{\partial s_{Sdt}}{\partial s_{SGt}} - \frac{\widetilde{\kappa}'(s_{Sct})}{(1+\widetilde{\kappa}(s_{Sct}))} \frac{\partial s_{Sct}}{\partial s_{SGt}}\right) Y_{SGt} \end{array}\right) \\
& + \frac{\left(1+\widetilde{\kappa}\left(s_{Sdt+1}\right)\right) \widetilde{\kappa}'\left(s_{Sdt}\right)^{1-\gamma} \mu_{Sdt+1} A_{Sdt}}{\left(1+\widetilde{\kappa}\left(s_{Sdt}\right)\right)} \frac{\partial s_{Sdt}}{\partial s_{SGt}} + \frac{\left(1+\widetilde{\kappa}\left(s_{Sct+1}\right)\right)^{1-\gamma} \mu_{Sct+1} \widetilde{\kappa}'\left(s_{Sct}\right) A_{Sct}}{\left(1+\widetilde{\kappa}\left(s_{Sct}\right)\right)} \frac{\partial s_{Sct}}{\partial s_{SGt}} \\
& - \frac{\left(1+\widetilde{\kappa}\left(s_{SHt+1}\right)\right)^{1-\gamma} \mu_{SHt+1} \widetilde{\kappa}'\left(s_{SHt}\right) A_{SHt}}{\left(1+\widetilde{\kappa}\left(s_{SHt}\right)\right)} \\
& + (1-\gamma)(\varepsilon-1) \left(\begin{array}{c} \frac{\widetilde{\kappa}'(s_{Sct})}{1+\widetilde{\kappa}(s_{Sct})} \frac{\partial s_{Sct}}{\partial s_{SGt}} \\ -\frac{\widetilde{\kappa}'(s_{Sdt})}{1+\widetilde{\kappa}(s_{Sdt})} \frac{\partial s_{Sdt}}{\partial s_{SGt}} \end{array}\right) \left(\begin{array}{c} \frac{\widetilde{\kappa}'\left(s_{Sd(t+1)}\right) \mu_{Sd(t+1)} A_{Sd(t+1)}}{1+\widetilde{\kappa}\left(s_{Sd(t+1)}\right)} \\ -\frac{\widetilde{\kappa}'\left(s_{Sc(t+1)}\right) \mu_{Sc(t+1)} A_{Sc(t+1)}}{1+\widetilde{\kappa}\left(s_{Sc(t+1)}\right)} \end{array}\right) \frac{A_{Sct}^{\varepsilon-1}}{A_{Sdt}^{\varepsilon-1}} \frac{\partial s_{Sd(t+1)}}{\partial a_t}
\end{aligned}
$$

Finally plugging this last equation into (113), one gets (95).

11.4.2 Appendix D.4.2 Other formula for the optimal tariff

From (28) one can define (at given technological levels), the relative price in the South as a function of imports in the North:

$$
p_t = \frac{\frac{\partial C_S}{\partial C_{SG}} \left(Y_{SGt}\left(p_t\right) - M_{Gt}, Y_{SHt}\left(p_t\right) - M_{Ht}\right)}{\frac{\partial C_S}{\partial C_{SH}} \left(Y_{SGt}\left(p_t\right) - M_{G}, Y_{SHt}\left(p_t\right) - M_{Ht}\right)},
$$

with partial derivatives:

$$
\frac{\partial p_t}{\partial M_G} = \frac{-\frac{\partial}{\partial C_G} \frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial C_S}{\partial C_{SH}}}}{\left(1 - \frac{\partial y_{SGt}}{\partial p_t} \frac{\partial}{\partial C_G} \frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial C_S}{\partial C_{SH}}} - \frac{\partial y_{SHt}}{\partial p_t} \frac{\partial}{\partial C_H} \frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial C_S}{\partial C_{SH}}}\right)}, \qquad (121)
$$

and

$$
\frac{\partial p_t}{\partial M_H} = \frac{-\frac{\partial}{\partial C_H} \frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial u}{\partial C_{SH}}}}{\left(1 - \frac{\partial y_{SGt}}{\partial p_t} \frac{\partial}{\partial C_G} \frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial C_S}{\partial C_{SH}}} - \frac{\partial y_{SHt}}{\partial p_t} \frac{\partial}{\partial C_H} \frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial C_S}{\partial C_{SH}}}\right)}. \qquad (122)
$$

Introducing this notation and rearranging terms in (94) yields

$$\frac{1}{p_t}\left(\frac{\partial C_N}{\partial C_{NG}}-\frac{\partial C_S}{\partial C_{SGt}}+\frac{\partial C_N}{\partial C_{NH}}b_t\frac{\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}{\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}\right)\left(M_{Gt}\frac{\partial p_t}{\partial M_G}+p_t\right) \tag{123}$$

$$=\left(-\frac{\omega_t\xi}{\lambda_t}\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}+\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_G}+\frac{\partial C_N}{\partial C_{NG}}-\frac{\partial C_S}{\partial C_{SGt}}.$$

Similarly rearranging terms in (111) gives:

$$\left(\frac{\partial C_N}{\partial C_{NH}}-\frac{\partial C_S}{\partial C_{SHt}}+\frac{\partial C_N}{\partial C_{NHt}}b_t\frac{\frac{\partial}{\partial C_H}\frac{\frac{\partial C_S}{\partial C_{SG}}}{\frac{\partial u}{\partial C_{SH}}}}{\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}\right)\left(1+M_{Gt}\frac{\partial p_t}{\partial M_H}\right)$$

$$=\left(\frac{\phi_t}{\lambda_t}\frac{\partial s_{SGt}}{\partial p_t}-\frac{\omega_t\xi}{\lambda_t}\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}\right)\frac{\partial p_t}{\partial M_H}+\frac{\partial C_N}{\partial C_{NH}}-\frac{\partial C_S}{\partial C_{SHt}}.$$

Taking the ratio of these two equations gives (34).

11.5 Appendix D.5 Case of maximizing (2).

I now look at the case of a social planner that maximizes North's welfare only. The solution for the North is similar to the solution in appendix B.8 but λ_t in these equations must be replaced by:

$$\lambda_{Nt}=\Psi\frac{\nu\left(S_t\right)^{1-\eta}}{\left(1+\rho\right)^t}C_{Nt}^{-\eta},$$

and the definition of ω_t is now

$$\omega_t=\frac{\nu'\left(S_t\right)\nu\left(S_t\right)^{-\eta}}{\left(1+\rho\right)^t}\left(\Psi C_{Nt}^{1-\eta}+\left(1-\Psi\right)C_{St}^{1-\eta}\right)+\left(1-I_{S_t>0}\Delta\right)\omega_{t+1},$$

instead of (74).

For the South part, the first order condition with respect to C_{St} gives:

$$\lambda_{St}=\left(1-\psi\right)\frac{\nu\left(S_t\right)^{1-\eta}}{\left(1+\rho\right)^t}C_{St}^{-\eta}.$$

(85) and (86) now write as:

$$\theta_{SHt}+\kappa_t\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}=\lambda_{St}\frac{\partial C_S}{\partial C_{SHt}}\text{ and }\theta_{SGt}+\kappa_t\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}=\lambda_{St}\frac{\partial C_S}{\partial C_{SGt}}. \tag{124}$$

(88), (87), (89), (90), (91) still hold, delivering again (93). Using (79), (71) and (72) for the North - replacing θ_{Gt} by θ_{NGt}- and (92), which still all hold, and (124) now gives:

$$\frac{M_{Gt}}{p_t}\left(\frac{\partial C_N}{\partial C_{NGt}}-\frac{\lambda_{St}}{\lambda_{Nt}}\frac{\partial C_S}{\partial C_{SGt}}\right) \tag{125}$$
$$= b_t\frac{\partial C_N}{\partial C_{NHt}}\left(p_t\frac{\partial y_{SG}}{\partial p_t}+\frac{\left(1-\frac{M_{Gt}}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}\right)}{\left(\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}\right)}\right)+\frac{1}{\lambda_{Nt}}\left(\phi_t\frac{\partial s_{SGt}}{\partial p_t}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}\right)$$

This equation shows that the optimum can be decentralized through a tariff. As before, I characterize this tariff further.

Following the same method as before, one can now get:

$$-M_{Ht}\left(\frac{\partial C_N}{\partial C_{NHt}}-\frac{\lambda_{St}}{\lambda_{Nt}}\frac{\partial C_S}{\partial C_{SHt}}\right) \tag{126}$$
$$= b_t\frac{\partial C_N}{\partial C_{NHt}}\left(-p_t\frac{\partial y_{SH}}{\partial p}+\frac{p_t+M_{Ht}\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}{\frac{\partial}{\partial C_{SHt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}-\frac{1}{p_t}\frac{\partial}{\partial C_{SGt}}\frac{\frac{\partial C_S}{\partial C_{SGt}}}{\frac{\partial C_S}{\partial C_{SHt}}}}\right)+\frac{p_t}{\lambda_{Nt}}\left(\phi_t\frac{\partial s_{SGt}}{\partial p_t}-\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}\right).$$

Combining these two equations with (121) and (122) and taking the ratio delivers the general version of (34).

Using (105), (104), (106), (107), (108) and (26), in (125) and (126) and taking the ratio (using (27) and (28)) gives:

$$\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}\left(p_t\left(1+b_t\right)\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)}b_tp_t\frac{\partial y_{SG}}{\partial p} \tag{127}$$
$$+\left(\begin{array}{c}\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{1+b_t}\frac{(1+b_t)\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}(1+\sigma b_t)}{\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}}\\-\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\end{array}\right)\frac{\nu^{\sigma}Y_{SHt}}{p_t\left(\nu^{\sigma}+p_t^{\sigma-1}(1-\nu)^{\sigma}\right)}$$
$$+\left(\begin{array}{c}\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\\-\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)}\frac{\nu^{\sigma}(1+(1-\sigma)b_t)+p_t^{\sigma-1}(1-\nu)^{\sigma}}{\nu^{\sigma}+p_t^{\sigma-1}(1-\nu)^{\sigma}}\end{array}\right)\frac{(1-\nu)^{\sigma}p_t^{\sigma-1}Y_{SGt}}{\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}}$$
$$=\frac{p_t}{\lambda_{Nt}}\left(\omega_t\xi\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\frac{\partial y_{SG}}{\partial p_t}-\phi_t\frac{\partial s_{SGt}}{\partial p_t}\right)$$
$$+\left(1-\frac{\lambda_{St}}{\lambda_{Nt}}\right)\left(\frac{C_{SHt}}{C_{SGt}}\frac{Y_{SGt}}{Y_{SHt}}-1\right)\frac{\nu^{\sigma}Y_{SHt}}{p_t}\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}-1} \tag{128}$$

This expression is identical to (113), except for the last term that indicates the terms of trade effects. When the social value of consumption is higher in the North than in the South

$(\lambda_{St} < \lambda_{Nt})$, this term pushes towards a positive trade tax if the South exports the polluting good $\frac{C_{SHt}}{C_{SGt}}\frac{Y_{SGt}}{Y_{SHt}} > 1$ (that is a tariff) and a negative one otherwise (that is an export tax).

To derive ϕ_t in this context, one can follow the same steps as in appendix D.4 up to (118) which still holds. Combining (124), (87), (105), (104), (106), (107), (92), (28), (108) and (90), one can express express θ_{SGt} and θ_{SHt} as:

$$\theta_{SGt} = \lambda_{St}\left(\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}} + \lambda_{Nt}\frac{b_t(1-\nu)^{\sigma} p_t^{\sigma-1}\left(\nu^{\sigma} + (1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{(1+b_t)\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)}, \tag{129}$$

$$\theta_{SHt} = \lambda_{St}\frac{\left(\nu^{\sigma} + (1-\nu)^{\sigma} p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{p_t} - \lambda_{Nt}\frac{b_t\nu^{\sigma}\left(\nu^{\sigma} + (1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{p_t(1+b_t)\left(\nu^{\sigma}+(1-\nu)^{\sigma} p_t^{\sigma-1}\right)}, \tag{130}$$

which replace (86) and (85). Using (88), one can then express:

$$\frac{\lambda_{SGt}}{\lambda_{SHt}} = p_t\left(\begin{array}{c} 1 + \frac{b_t}{(1+b_t)}\left(\frac{\lambda_{St}}{\lambda_{Nt}}\frac{\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{\left(\nu^{\sigma}+(1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}} - \frac{\nu^{\sigma}}{\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}}\frac{b_t}{(1+b_t)}\right)^{-1} \\ -\frac{\omega_t\xi}{\lambda_{Nt}}\left(\frac{A_{Sdt}}{A_{SGt}}\right)^{\varepsilon}\left(\frac{\lambda_{St}}{\lambda_{Nt}}\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}} - \frac{\nu^{\sigma}\left(\nu^{\sigma}+(1-\nu)^{\sigma}(p_t(1+b_t))^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}{\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}}\frac{b_t}{(1+b_t)}\right)^{-1} \end{array}\right).$$

Plugging this last equation with (129), (130) and (82) in (118) exactly delivers (120) with λ_t replaced by λ_{Nt}. Plugging this modified equation (120) in (127) delivers (95), with λ_{Nt} replacing λ_{St} on the right hand side and the additional term

$$\left(1 - \frac{\lambda_{St}}{\lambda_{Nt}}\right)\left(\frac{C_{SHt}}{C_{SGt}}\frac{Y_{SGt}}{Y_{SHt}} - 1\right)\frac{\nu^{\sigma}Y_{SHt}}{p_t}\left(\nu^{\sigma}+(1-\nu)^{\sigma}p_t^{\sigma-1}\right)^{\frac{1}{\sigma-1}-1}$$

in the right-hand side. This equation has the form described in subsection 4.2. The second term which involves the difference in the social value of future innovation in sector H and sector G in the South has a potentially ambiguous sign. Future environmental costs of sector G innovation in the South still push towards a positive term, but the impact on future terms of trade could push towards a positive or a negative trade tax.

11.6 Appendix D.6 Proof of proposition 5

First I deal with the case $\eta \leq 1$ and second with the case $\sigma < 1$ and $\eta > 1$. Note that if the optimal policy avoids a disaster, part 2 of Appendix B.9 still holds and all resources (capital, labor, scientists) devoted to sector G in the South must tend towards 0 and A_{SHt} must grow at a rate faster than $(1+\kappa)^{1-\gamma} - 1 - x$ for any $x > 0$. Finally I show that when $\Psi = 1$, there is never full specialization in the South (except if the South is just at a corner of specialization).

11.6.1 Case $\eta \leq 1$.

The proof follows very closely Appendix B.7 (and A.9). Here as well, part 1 follows provided that avoiding the disaster is possible. For part 2 the second best policy is still feasible. Under that policy both countries are fully specialized, so that in both countries the asymptotic growth rate is given by $(1+\kappa)^{1-\gamma}-1$. In part 3 it is still true that the North must then asymptotically innovate in clean technologies only (and the South in sector H) - otherwise world consumption and therefore North consumption could not grow at the maximal growth rate. The remaining no longer holds though.

11.6.2 Case $\eta > 1$ and $\sigma < 1$

As already mentioned part 2 remains true, but so do parts 1 and 3 here. Part 4 holds with the following changes: The first paragraph is unnecessary since now I know that the South does not fully specialize already. The second paragraph shows that the North switches to exporting good G. The rest of the analysis holds without any change: a positive mass of scientists must be allocated to clean technologies in the North.

11.6.3 The South never fully specialize beyond the corner when $\Psi = 1$

Assume that the South is fully specialized in sector H and not at a corner of specialization (that is $\left(p_t \frac{A_{SGt}}{A_{SHt}}\right)^{\frac{1}{\alpha-\beta}} < \left(\frac{\beta^\alpha(1-\beta)^{(1-\alpha)}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{L_S}{K_S}$). Then $C_{SGt} = -M_{Gt}$, $\frac{\partial y_{SG}}{\partial p_t} = 0$ and $\frac{\partial s_{SGt}}{\partial p_t} = 0$, so that (127) for $\Psi = 1$ leads to: $b_t = -\frac{\left(\nu^\sigma + (1-\nu)^\sigma p_t^{\sigma-1}\right)}{\nu^\sigma + \sigma(1-\nu)^\sigma p_t^{\sigma-1}} \leq -1$ with $\sigma \leq 1$, which is absurd: this case is impossible. Combining with lemma 3, the South must therefore move towards the corner of full specialization.

11.7 Appendix D.7: Knowledge spillovers

In this appendix, I first derive the $\limsup$ and the $\liminf$ of the ratio of productivities across countries in the long-run, second I prove proposition 6. Third, I show that unless scientists in the South all asymptotically innovate in the dirty sector, the North can ensure a switch in the South too. Forth I study perfect substitute case.

11.7.1 Part 1: Limits on the ratio of productivities:

Assume that $\frac{A_{Xzt}}{A_{(-X)zt}} > (1+\kappa)^{\frac{1-\gamma}{\delta}}$, since $\frac{\overline{A}_{Xz(t+1)}}{\overline{A}_{(-X)z(t+1)}} = \left(\frac{A_{Xzt}}{A_{(-X)zt}}\right)^{1-\delta}$, one gets:

$$\frac{A_{Xz(t+1)}}{A_{(-X)z(t+1)}} \leq (1+\kappa)^{1-\gamma} \frac{\overline{A}_{Xz(t+1)}}{\overline{A}_{(-X)z(t+1)}} < \frac{A_{Xzt}}{A_{(-X)zt}},$$

so that even if all innovation in country X occurs in sector z and no innovation in country $(-X)$ does, the ratio of productivity decreases over time. Therefore, regardless of the pattern of innovation:

$$\liminf \frac{A_{Xzt}}{A_{(-X)zt}} \geq (1+\kappa)^{\frac{\gamma-1}{\delta}} \text{ and } \limsup \frac{A_{Xzt}}{A_{(-X)zt}} \leq (1+\kappa)^{\frac{1-\gamma}{\delta}},$$

for $z \in \{c, d, H\}$, and as this holds for $z \in \{c, d, H\}$, it also holds for $z = G$.

11.7.2 Part 2: Proof of proposition 6

If initial technologies are sufficiently close to each other, κ is sufficiently small and the spillovers δ are sufficiently strong, technologies are going to remain close to each other all along the equilibrium path. In particular the South keeps its comparative advantage in the polluting sector. Following the logic of Appendix B.7 ii), more scientists will initially be allocated to sector G in the South than in the North. If $A_{Sc(t-1)}/A_{Sd(t-1)}$ is sufficiently small, nearly all of these scientists will be allocated to the dirty subsector, therefore even if all sector G scientists in the North gets allocated to the clean sector, A_{Sdt} grows faster than A_{Ndt}, and the ratio A_{Sct}/A_{Sdt} remains small: a disaster is unavoidable.

11.7.3 Part 3: South switches to clean unless all its scientists asymptotically innovate in dirty

In the absence of any policy in the South, the equilibrium will feature allocating more scientists towards clean intermediates if (see (63)) $\overline{A}_{Sct}^{\varepsilon-1} > \overline{A}_{Sdt}^{\varepsilon-1}$. If the North implements sufficiently large clean research subsidies, it can ensure that A_{Nct} and therefore $\overline{A}_{Sct}$ grows faster than $\left((1+\kappa)^{1-\gamma} - x\right)^t$, no matter how small $x\,(>0)$ is. In the meantime, the only source of improvements in $\overline{A}_{Sdt}$ comes from innovation in the South, so unless $\overline{A}_{Sdt}$ also grows faster than $\left((1+\kappa)^{1-\gamma} - x\right)^t$, for any $x > 0$, $\overline{A}_{Sct}$ will eventually get larger than $(1+\kappa)^{|(\varepsilon-1)(1-\gamma)-1|}\,\overline{A}_{Sdt}^{\varepsilon-1}$, leading to a switch in the South towards mostly clean innovation. A necessary condition to get $\overline{A}_{Sdt}$ dominating $\left((1+\kappa)^{1-\gamma} - x\right)^t$ is to have $\limsup s_{dt} = 1$.

11.7.4 Part 4: Perfect substitute case

Here, I formally study the perfect substitute case

Remark 4 Assume that final consumption is Cobb-Douglas in the polluting and non-polluting goods ($\sigma = 1$) and that clean and dirty inputs are perfect substitutes ($\varepsilon = \infty$). First, i) if initial relative endowments are sufficiently close to each other ($\frac{1-\alpha}{\alpha}\frac{K_S}{L_S} < \frac{1-(\alpha\nu+\beta(1-\nu))}{\alpha\nu+\beta(1-\nu)}\frac{(1+\kappa)^{-\frac{1-\gamma}{\delta}}K_N+K_S}{(1+\kappa)^{\frac{1-\gamma}{\delta}}L_N+L_S} <$

$(1+\kappa)^{-2\frac{1-\gamma}{\delta}}\frac{1-\beta}{\beta}\frac{K_N}{L_N}$ is a sufficient condition) and the initial environmental quality is sufficiently large, then temporary clean research subsidies in the North alone can prevent a disaster. Second, ii) if the South has a sufficiently large capital labor ratio ($\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}K_N^{\beta}L_N^{1-\beta} > (1+\kappa)^{\frac{1-\gamma}{\delta}}\frac{1-\nu}{\nu}K_S^{\beta}L_S^{1-\beta}$ and $(1+\kappa)^{\frac{1-\gamma}{\delta}}K_N^{\alpha}L_N^{1-\alpha} < \frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\frac{1-\nu}{\nu}K_S^{\alpha}L_S^{1-\alpha}$), technologies are sufficiently close across countries ($A_{Nz0}/A_{Sz0} \in \left((1+\kappa)^{-\frac{1-\gamma}{\delta}}, (1+\kappa)^{\frac{1-\gamma}{\delta}}\right)$ for $z \in \{c, d, H\}$), and clean technologies are less advanced than dirty ones in the South ($A_{Sc0} < A_{Sd0}(1+\kappa)^{-(1-\gamma)\left(1+\frac{1}{\delta}\right)}$), then, clean research subsidies alone can never prevent a disaster.

Proof of point i) I assume that the North innovates in clean only. There is perfect substitutability, so the South innovates in dirty only as long $\overline{A_{Sdt}} > (1+\kappa)^{1-\gamma}\overline{A_{Sct}}$ (there can be multiple equilibria $\overline{A_{Sct}} < \overline{A_{Sdt}} < (1+\kappa)^{1-\gamma}\overline{A_{Sct}}$ in the perfect substitute case). (53) and (54) can be written as:

$$\frac{\widetilde{K_X}}{\widetilde{L_X}} > \frac{\beta\left(\nu\left(1-\alpha\right)+\left(1-\beta\right)\left(1-\nu\right)\right)}{\left(1-\beta\right)\left(\alpha\nu+\beta\left(1-\nu\right)\right)}\frac{\widetilde{K_N}+\widetilde{K_S}}{\widetilde{L_N}+\widetilde{L_S}}, \tag{131}$$

$$\frac{\widetilde{K_X}}{\widetilde{L_X}} < \frac{\alpha\left(\left(1-\alpha\right)\nu+\left(1-\beta\right)\left(1-\nu\right)\right)}{\left(1-\alpha\right)\left(\alpha\nu+\beta\left(1-\nu\right)\right)}\frac{\widetilde{K_N}+\widetilde{K_S}}{\widetilde{L_N}+\widetilde{L_S}}. \tag{132}$$

Assume that

$$\begin{aligned}\frac{K_S}{L_S} &< \frac{\alpha}{1-\alpha}\frac{1-\left(\alpha\nu+\beta\left(1-\nu\right)\right)}{\alpha\nu+\beta\left(1-\nu\right)}\frac{(1+\kappa)^{-\frac{1-\gamma}{\delta}}K_N+K_S}{(1+\kappa)^{\frac{1-\gamma}{\delta}}L_N+L_S},\\ \frac{K_N}{L_N} &> \frac{(1+\kappa)^{2\frac{1-\gamma}{\delta}}\beta}{1-\beta}\frac{1-\left(\alpha\nu+\beta\left(1-\nu\right)\right)}{\alpha\nu+\beta\left(1-\nu\right)}\frac{(1+\kappa)^{-\frac{1-\gamma}{\delta}}K_N+K_S}{(1+\kappa)^{\frac{1-\gamma}{\delta}}L_N+L_S},\end{aligned}$$

then either $\frac{\widetilde{K_N}}{\widetilde{L_N}} \geq \frac{\widetilde{K_S}}{\widetilde{L_S}}$ or if $\frac{\widetilde{K_N}}{\widetilde{L_N}} \leq \frac{\widetilde{K_S}}{\widetilde{L_S}}$, eventually (131) and (132) must be satisfied, therefore the South specializes in good H or produces both goods. When the South specializes in the non-polluting good, $s_{Sdt} = 0$. When the South does not specialize, the allocation of research in the South follows (64), that is:

$$\frac{\widetilde{\kappa}'\left(s_{SGt}\right)}{1+\widetilde{\kappa}\left(s_{SGt}\right)}\frac{1+\widetilde{\kappa}\left(s_{SHt}\right)}{\widetilde{\kappa}'\left(s_{SHt}\right)} = \frac{\nu}{1-\nu}\frac{Y_{SHt}}{Y_{NHt}+Y_{SHt}}\frac{Y_{NGt}+Y_{SGt}}{Y_{SGt}}.$$

Since I assumed that innovation occurs in dirty in the South. Therefore, the only way to ensure $\limsup s_{dt} = 1$, is to have $\liminf \frac{Y_{SHt}}{Y_{NHt}+Y_{SHt}} = 0$, since technologies in the North and the South converge towards each other, this is possible only if $\frac{\widetilde{K_S}}{\widetilde{L_S}} \to \frac{\alpha(1-\alpha)\nu+\alpha(1-\beta)(1-\nu)}{\alpha(1-\alpha)\nu+\beta(1-\alpha)(1-\nu)}\frac{\widetilde{K_N}+\widetilde{K_S}}{\widetilde{L_N}+\widetilde{L_S}}$, which is impossible with $\frac{K_S}{L_S} < \frac{\alpha}{1-\alpha}\frac{1-(\alpha\nu+\beta(1-\nu))}{\alpha\nu+\beta(1-\nu)}\frac{(1+\kappa)^{-\frac{1-\gamma}{\delta}}K_N+K_S}{(1+\kappa)^{\frac{1-\gamma}{\delta}}L_N+L_S}$.

Proof of point ii) If initially technologies are close to each other: $(A_{Nz0}/A_{Sz0} \in \left((1+\kappa)^{-\frac{1-\gamma}{\delta}}, (1+\kappa)^{\frac{1-\gamma}{\delta}}\right))$ and if:

$$\frac{\alpha^{\beta}(1-\alpha)^{1-\beta}}{\beta^{\beta}(1-\beta)^{(1-\beta)}} K_N^{\beta} L_N^{1-\beta} \geq (1+\kappa)^{\frac{1-\gamma}{\delta}} \frac{1-\nu}{\nu} K_S^{\beta} L_S^{1-\beta}$$

and

$$(1+\kappa)^{\frac{1-\gamma}{\delta}} K_N^{\alpha} L_N^{1-\alpha} \leq \frac{\beta^{\alpha}(1-\beta)^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}} \frac{1-\nu}{\nu} K_S^{\alpha} L_S^{1-\alpha}, \tag{133}$$

then regardless of innovation and any potential carbon tax (57) and (58) are satisfied, so that both countries are fully specialized and innovation in the South occurs in the polluting sector only. If $A_{Sc0}/A_{Sd0} < (1+\kappa)^{-\frac{(1-\gamma)(\delta+1)}{\delta}}$, then considering that initially $A_{Sct}/A_{Nct} > (1+\kappa)^{\frac{1-\gamma}{\delta}}$, every period $\overline{A}_{Sct}/\overline{A}_{Sdt} < (1+\kappa)^{-(1-\gamma)}$ and innovation occurs in dirty technologies only in the South.

11.8 Appendix D.8: Proof of proposition 7

Assume that the subsidy to the use of all intermediates is the same in both countries (this is important since scale will now matter). Solving for the profit maximization problem of monopolists in the three subsectors, the allocation of scientists is characterized by the following equations:

$$\frac{\omega_N}{1-\gamma} = \widetilde{\kappa}'(s_{NHt})\left(\frac{p_{Ht}Y_{NHt}}{1+\widetilde{\kappa}(s_{NHt})+\widetilde{\kappa}(s_{SHt})} + \frac{p_{Ht}Y_{SHt}}{1+\widetilde{\kappa}(s_{NHt})+\widetilde{\kappa}(s_{SHt})}\right),$$

$$\frac{\omega_N}{1-\gamma} = \frac{\widetilde{\kappa}'(s_{Ndt})}{1+q_{Ndt}}\left(\frac{(1+\tau_{Nt})^{-\varepsilon}A_{dt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+(1+\tau_{Nt})^{1-\varepsilon}A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{NGt}}{1+\widetilde{\kappa}(s_{Ndt})+\widetilde{\kappa}(s_{Sdt})} + \frac{A_{dt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{SGt}}{1+\widetilde{\kappa}(s_{Ndt})+\widetilde{\kappa}(s_{Sdt})}\right),$$

$$\frac{\omega_N}{1-\gamma} = \widetilde{\kappa}'(s_{Nct})\left(\frac{A_{ct}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+(1+\tau_{Nt})^{1-\varepsilon}A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{NGt}}{1+\widetilde{\kappa}(s_{Nct})+\widetilde{\kappa}(s_{Sct})} + \frac{A_{ct}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{SGt}}{1+\widetilde{\kappa}(s_{Nct})+\widetilde{\kappa}(s_{Sct})}\right),$$

$$\frac{\omega_S}{1-\gamma} = \widetilde{\kappa}'(s_{SHt})\left(\frac{p_{Ht}Y_{NHt}}{1+\widetilde{\kappa}(s_{NHt})+\widetilde{\kappa}(s_{SHt})} + \frac{p_{Ht}Y_{SHt}}{1+\widetilde{\kappa}(s_{NHt})+\widetilde{\kappa}(s_{SHt})}\right),$$

$$\frac{\omega_S}{1-\gamma} = \widetilde{\kappa}'(s_{Sdt})\left(\frac{(1+\tau_{Nt})^{-\varepsilon}A_{dt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+(1+\tau_{Nt})^{1-\varepsilon}A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{NGt}}{1+\widetilde{\kappa}(s_{Ndt})+\widetilde{\kappa}(s_{Sdt})} + \frac{A_{dt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{SGt}}{1+\widetilde{\kappa}(s_{Ndt})+\widetilde{\kappa}(s_{Sdt})}\right),$$

$$\frac{\omega_S}{1-\gamma} = \widetilde{\kappa}'(s_{Sct})\left(\frac{A_{ct}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+(1+\tau_{Nt})^{1-\varepsilon}A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{NGt}}{1+\widetilde{\kappa}(s_{Nct})+\widetilde{\kappa}(s_{Sct})} + \frac{A_{ct}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}\frac{p_{Gt}Y_{SGt}}{1+\widetilde{\kappa}(s_{Nct})+\widetilde{\kappa}(s_{Sct})}\right).$$

Case of the carbon tax only

When the dirty research tax is null, the equations are identical between the North and the South. There will be less scientists allocated to the clean subsector than to the dirty subsector if and only if:

$$\frac{(1+\tau_{Nt})^{-\varepsilon}A_{dt}^{\varepsilon-1}Y_{NGt}}{A_{ct}^{\varepsilon-1}+(1+\tau_{Nt})^{1-\varepsilon}A_{dt}^{\varepsilon-1}} + \frac{A_{dt}^{\varepsilon-1}Y_{SGt}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}} > \frac{A_{ct}^{\varepsilon-1}Y_{NGt}}{A_{ct}^{\varepsilon-1}+(1+\tau_{Nt})^{1-\varepsilon}A_{dt}^{\varepsilon-1}} + \frac{A_{ct}^{\varepsilon-1}Y_{SGt}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}.$$

The larger the tax rate, the less likely it is that this inequality holds, therefore there will be less clean innovation than dirty for any tax rate (at given production levels) if:

$$\frac{A_{dt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}Y_{SGt} > Y_{NGt} + \frac{A_{ct}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}Y_{SGt},$$

this equality is necessarily satisfied for sufficiently low A_{c0}/A_{d0} if $Y_{SGt} > Y_{NGt}$ (A_{c0}/A_{d0} sufficiently low combined with the assumption that there is more dirty than clean innovations implies that A_{ct}/A_{dt} is sufficiently small every period). In fact Y_{SGt}/Y_{NGt} is necessarily larger with a carbon tax than without, so a sufficient condition is that $Y_{SGt} > Y_{NGt}$ when there is no carbon tax.

Without a carbon tax and when $\sigma = 1$, I can compute equilibrium prices following (52), in the case where there is not specialization, as:

$$\left(\frac{p_{Gt}}{p_{Ht}}\frac{A_{Gt}}{A_{Ht}}\right)^{\frac{1}{\alpha-\beta}} = \left(\frac{\beta^{\beta}(1-\beta)^{(1-\beta)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right)^{\frac{1}{\alpha-\beta}} \frac{\alpha\nu+\beta(1-\nu)}{(1-\alpha)\nu+(1-\beta)(1-\nu)}\frac{(L_N+L_S)}{(K_N+K_S)}. \quad (134)$$

The equilibrium production levels are given by:

$$\begin{aligned} Y_{NGt} &= \frac{\zeta A_{Gt}}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}} \\ &\times\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_{Gt}}{p_{Ht}}\frac{A_{Gt}}{A_{Ht}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_N-\beta\left(\frac{p_{Gt}}{p_{Ht}}\frac{A_{Gt}}{A_{Ht}}\right)^{-\frac{\alpha}{\alpha-\beta}}L_N\right), \end{aligned}$$

$$\begin{aligned} Y_{SGt} &= \frac{\zeta A_{Gt}}{(\alpha-\beta)}\left(\frac{\beta^{\beta\alpha}(1-\beta)^{(1-\beta)\alpha}}{\alpha^{\beta\alpha}(1-\alpha)^{(1-\alpha)\beta}}\right)^{\frac{1}{\alpha-\beta}} \\ &\times\left(\left(\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\beta^{\beta}(1-\beta)^{(1-\beta)}}\right)^{\frac{1}{\alpha-\beta}}(1-\beta)\left(\frac{p_{Gt}}{p_{Ht}}\frac{A_{Gt}}{A_{Ht}}\right)^{\frac{1-\alpha}{\alpha-\beta}}K_S-\beta\left(\frac{p_{Gt}}{p_{Ht}}\frac{A_{Gt}}{A_{Ht}}\right)^{-\frac{\alpha}{\alpha-\beta}}L_S\right), \end{aligned}$$

therefore the ratio of productivity levels without the carbon tax is independent of technology levels: if $Y_{SG0} \geq Y_{NG0}$, then $Y_{SGt|\tau_{Nt}=0} \geq Y_{NGt|\tau_{Nt}=0}$ for all t, and $Y_{SGt|\tau_{Nt}\neq 0} > Y_{NGt|\tau_{Nt}\neq 0}$.

It is direct to check that when $\sigma = 1$, and $\tau_{Nt} = 0$, the conditions for specialization are also independent of the technology levels, and that when one country specializes in good G but not the other one, the ratio of production of good G is still independent of technology levels.

Combining the carbon tax with the tax on dirty research

When the carbon tax is combined with clean research subsidies, more scientists will be allocated to dirty research in the South than to clean research in both countries if $\frac{A_{Sdt}^{\varepsilon-1}}{A_{Sct}^{\varepsilon-1}+A_{Sdt}^{\varepsilon-1}}Y_{SGt}$ is sufficiently large relative to $Y_{NGt} + \frac{A_{Sct}^{\varepsilon-1}}{A_{Sct}^{\varepsilon-1}+A_{Sdt}^{\varepsilon-1}}Y_{SGt}$. Indeed in that case, even for arbitrarily large carbon taxes, s_{Sct} becomes arbitrarily small, so that the allocation of scientists in the South will be given by

$$\begin{aligned}&\frac{\widetilde{\kappa}'(s_{Sdt})}{1+\widetilde{\kappa}(s_{Sdt})}p_{Gt}\left(\frac{(1+\tau_{Nt})^{-\varepsilon}A_{dt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{dt}\right)^{\varepsilon-1}}Y_{NGt}+\frac{A_{dt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}Y_{SGt}\right)\\ =\ &\widetilde{\kappa}'(1-s_{Sdt})\frac{p_{Ht}(Y_{NHt}+Y_{SHt})}{1+\widetilde{\kappa}(s_{NHt})+\widetilde{\kappa}(s_{SHt})},\end{aligned}$$

while the allocation of scientists in the North is given by

$$\begin{aligned}&\frac{\widetilde{\kappa}'(s_{Nct})}{1+\widetilde{\kappa}(s_{Nct})}p_{Gt}\left(\frac{A_{ct}^{\varepsilon-1}}{A_{Nct}^{\varepsilon-1}+\left((1+\tau_{Nt})^{-1}A_{Ndt}\right)^{\varepsilon-1}}Y_{NGt}+\frac{A_{ct}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}+A_{dt}^{\varepsilon-1}}Y_{SGt}\right)\\ =\ &\frac{\widetilde{\kappa}'(1-s_{Nct})}{1+\widetilde{\kappa}(s_{NHt})+\widetilde{\kappa}(s_{SHt})}p_{Ht}(Y_{NHt}+Y_{SHt}),\end{aligned}$$

- for a sufficiently large tax on dirty research which in a given period maximizes the number of scientists allocated to the clean sector in the North. Therefore $s_{Nct} < s_{Sdt}$, and with sufficiently small s_{Sct}, $\widetilde{\kappa}(s_{Nct}) + \widetilde{\kappa}(s_{Sct}) < \widetilde{\kappa}(s_{Sdt})$ and A_{dt} grows faster than A_{ct}. For sufficiently small A_{Sc0}/A_{Sd0}, $\frac{A_{Sdt}^{\varepsilon-1}}{A_{Sct}^{\varepsilon-1}+A_{Sdt}^{\varepsilon-1}}Y_{SGt} > Y_{NGt} + \frac{A_{Sct}^{\varepsilon-1}}{A_{Sct}^{\varepsilon-1}+A_{Sdt}^{\varepsilon-1}}Y_{SGt}$, provided that Y_{NG0}/Y_{SG0} is sufficiently small: since Y_{NGt}/Y_{SGt} before implementation of the carbon tax depends only on relative endowments, it is always equal to Y_{NG0}/Y_{SG0}, and smaller for a positive carbon tax.