The fossil Episode

John Hassler

IIES, Stockholm University

Hans-Werner Sinn

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

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Abstract

This article presents a two-sector intertemporal general equilibrium model of industrial development with a one-sided substitutability between fossil carbon and biocarbon that has four phases: (1) a pure biocarbon phase, (2) a pure fossil carbon phase, (3) a mixed fossil and biocarbon phase and (4) another biocarbon phase. The only exogenous shock generating the phases endogenously is the unforeseen invention of a technology ("the steam and combustion engines") that is able to make productive use of fossil carbon. If there is enough fossil carbon in the ground and food is essential enough, the four phases will necessarily take place, making the fossil carbon phases a mere historical intermezzo within a biocarbon eternity.

Introduction

The year 2007 may well have marked a turning point in the history of mankind, because it was the year of the Tortilla crisis. In January 2007, thousands of people protested against rising food prices in Mexico City. Corn, imported from the US, had become nearly twice as expensive as the previous winter, increasing the price of tortillas by 35%. A year later, wheat and rice prices had trebled, and the hunger riots had spread to 37 countries.[1]

Arguably this development was caused by the rapid increase in the production of bioethanol from corn in the US, and to a lesser extent by the rise of biodiesel in Europe and bioethanol in Latin America. While ethanol made from corn played hardly any role until the late 1970s, it became a dominant economic factor in US farming in the first decade of this century. By the time of the Tortilla crisis, 8% of US agricultural land or 30% of US total corn output was used for the production of bioethanol,[2] and in the fiscal year 2010/2011, nearly 40% of the US corn harvest was used for this purpose.[3] Almost the entire increase in the world's corn production from 2004 until 2007 ended up as input for the ballooning US bioethanol production.[4]

Interestingly, the increase in food prices coincided with the increase in oil prices and may even have been caused by it. After all, in the two years preceding the Tortilla crisis the price of crude oil had risen by two thirds. Oil prices even quadrupled between 2000 and 2008, and so did bioethanol production. According to a study by the International Food Policy Research Institute (IFPRI), biofuel production accounted for 30% of the rise in the average price of corn, rice and wheat between 2000 and 2007,[5] and according to Mitchell (2008), most of the price increase resulted from soaring oil prices. These findings triggered a heated debate. Authors like Piesse and Thirtle (2009), Headey and Fan (2008) or Collins

[1] See Sinn (2012, chapter 3, "Table or Tank").

[2] von Braun (2008).

[3] National Corn Growers Association (2008, 2012).

[4] World corn production rose during this period by 55 million tons, while the US consumption of corn for bioethanol production rose by 50 million tons. See Mitchell (2008).

[5] See Rosegrant (2008).

(2008) basically supported Mitchell. Gilbert (2010) and Ajanovic (2010) were more critical about causality, but even they did not deny the close substitutability of food and energy.

Following this debate we are inclined to see the Tortilla crisis as a turning point in history at which the fossil energy price reached the food price and led to a sudden linkage between the food and energy markets. For two hundred years, since the start of the Industrial Revolution when mankind learned to tap fossil fuel as a resource, fossil carbon had been a much cheaper source of energy than biocarbon. Its abundance may well have liberated about half of the land available from the production of fodder for drought and pack animals,[6] and made it possible to both enhance both general standards of nutrition and the energy input to industrial production, fueling an unprecedented growth process. However, over time, the exhaustible fossil fuel resource became scarcer, and the fossil fuel price continued to rise relative to the food price, eventually reaching that price, making it more attractive for farmers to deliver their crop to the filling station instead of the grocery store.

The linkage between the two markets and prices was sudden, as there is only a one-sided substitution possibility between fossil carbon and biocarbon. While the latter can be transformed into energy by simple chemistry, the reverse process is not available and if so, only at prohibitive cost. (You can put the vegetable oil from the grocery store into your diesel tank, but it is not advisable to add the oil you buy at the filling station to your salad.) Thus the two kinds of energy were separate as long as fossil carbon was cheaper than biocarbon, and became perfect substitutes after the fossil carbon price reached the biocarbon price and biocarbon became increasingly directed from the table to the tank.

In our opinion, mankind should now reconsider the possibility that, due to an increasing energy scarcity, the oil price will continue to rise in the foreseeable future, pulling up food prices and making it increasingly attractive for farmers to devote growing shares of their land to the production of biofuel.

This paper presents a formal intertemporal general equilibrium model with an industrial

[6] See Sinn (2012, p. 117)

sector operating with capital, labor and energy as inputs and an agricultural sector using land, capital and labor, to investigate the impact of the emergence and use of fossil carbon on the development of the world economy. With just one unforeseen shock, the appearance of a technical device to exploit the fossil fuel reservoirs, namely steam and combustion engines, we can endogenously derive a development path with four stages: (1) A pre-industrial stage where food and fodder rival for land and bio energy is the only form of energy available. (2) A fossil phase where land is exclusively used for the production of food and energy is only taken from fossil sources. (3) A mixed phase where biocarbon and fossil carbon are both used as a source of energy and an increasing share of land is gradually absorbed by bioenergy production. (4) A final bio phase where the stock of fossil carbon is exhausted and, as in the pre-industrial phase, biocarbon serves both nutrition and energy needs.

The model developed here is based on first principles such as rational expectations (or perfect foresight) and market clearing satisfying the main theorem of welfare economics. Thus, the allocation of resources it describes is Pareto optimal both in an intersectoral and an intertemporal sense satisfying, for example, the Hotelling rule or Solow-Stiglitz efficiency condition in phases (2) and (3). Admittedly, the model is extremely abstract. But it is powerful insofar as with only one exogenous shock, it is able to endogenously derive the transition through the four phases.

There are two reasons for setting the model up in this way. Firstly, it allows us to simplify our task of modeling the market economy because we can formulate the intertemporal general equilibrium as a central planning approach. Secondly, by assuming perfect foresight, we are able to insulate our findings from the influence of arbitrary expectation assumptions. As the stages of economic growth result endogenously from these simple assumptions, we are confident that variations in the assumptions that make the model both more complicated and realistic will not change the essence of our story.

A model of multiple use of biocarbon

We model a two-sector growth model. The first sector produces a consumption good using a production function

$$Y_t = A_{c,t} K_{1,t}^{\alpha_1} L_{1,t}^{1-\alpha_1-\nu_1} E_t^{\nu_1} \tag{1}$$

where Y_t is final output gross of depreciation, $A_{c,t}$ is an exogenous productivity trend, $K_{1,t}$ and $L_{1,t}$ are capital and labor used in sector 1 and E_t is energy input. The consumption good serves both consumption and investment purposes. The second sector is agriculture; it produces biocarbon F_t that also has two potential uses. It can be consumed directly and it can also be used as energy input to the manufacturing sector. Although we speak of an agricultural sector producing biocarbon for food or energy, sector 2 may be given a broader meaning, representing land-intensive production including e.g., wind energy. The production function of the agricultural sector is

$$F_t = A_{f,t} K_{2,t}^{\alpha_2} L_{2,t}^{(1-\alpha_2-\nu_2)} T^{\nu_2}. \tag{2}$$

$A_{f,t}$ is an exogenous productivity trend, $K_{2,t}$ and $L_{2,t}$ are capital and labor used in sector 2 and T is a fixed factor only used in sector 2. To fix ideas, we think of T as land that is used in the production of biocarbon.

Individuals maximize

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \theta \ln D_t\right) \tag{3}$$

where C_t is consumption of manufactured goods from sector one and D_t is consumption of food, i.e. biocarbon produced in sector two. D can, as mentioned above, have a broader interpretation. The parameter θ determines the relative taste for food (more exactly, the taste for goods from sector 2), so that $\frac{\theta}{1+\theta}$ is the share of income spent on food.

We denote the flow of biocarbon that is used as energy input in sector 1 by B_t. Sector

1's energy input can also come from oil, denoted by O_t, that is taken from a non-renewable and finite stock R_t without incurring extraction cost. B_t and O_t are perfect substitutes. It is important for our analysis that there is a one-sided substitutability between biocarbon and fossil carbon. While biocarbon can be used for energy services, fossil carbon cannot be eaten. As discussed in the introduction, we will model an intial phase 1, when the technology for using fossil energy has yet not developed. We show that this phase easily can be added to analysis after we have formally modelled the economy from the date the new technology is discovered.

In addition to (1) and (2) the economy faces the following constraints:

$$\begin{aligned} C_t + K_{t+1} &= Y_t \\ F_t &= D_t + B_t \\ E_t &= O_t + B_t \\ K_t &= K_{1,t} + K_{2,t} \\ L_t &= L_{1,t} + L_{2,t} \\ B_t &\geq 0 \\ R_0 &\geq \sum_{t=0}^{\infty} O_t \\ O_t &\geq 0 \end{aligned} \tag{4}$$

The first constraint is the resource constraint for goods from sector 1. Output is split between consumption and next period's capital stock. For analytical tractability, we assume that capital depreciates fully between periods. In our numerical examples, we will assume a period is 10 years, somewhat justifying the depreciation assumption.

The second equation states that output from agriculture (sector 2) is split between consumption and energy use. The third states that energy services stem from fossil carbon and that part of the biocarbon production that is not consumed as food. The fourth and fifth equations are, respectively, the aggregate resource constraints for capital and labor.

The sixth equation states that the use of biocarbon for energy has to be non-negative. This reflects our assumption that fossil fuel cannot be eaten. Finally, there are non-negativity constraints on the stocks and flows of fossil carbon.

We assume perfect markets so that the planning allocation is identical to the competitive equilibrium which is found by maximizing (3) subject to the production functions (1), (2) and the resource constraints (4).

The Kuhn-Tucker formulation of the planner's problem can be formulated as

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left(A_{c,t} \left(K_t - K_{2,t}\right)^{\alpha_1} \left(L_t - L_{2,t}\right)^{1-\alpha_1-\nu_1} \left(O_t + B_t\right)^{\nu_1} - K_{t+1} \right) + \theta \ln \left(F_t - B_t\right) \right.$$
$$\left. + \lambda_t^F \left(A_{f,t} (K_{2,t})^{\alpha_2} \left(L_{2,t}\right)^{(1-\alpha_2-\nu_2)} T^{\nu_2} - F_t \right) + \lambda_t^B B_t + \lambda_t^O O_t + \beta^{-t} \lambda^R \left(R_0 - \sum_0^{\infty} O_t \right) \right\}$$

The associated first order conditions are

$$\begin{aligned}
K_{t+1}; -\frac{1}{C_t} + \beta \frac{1}{C_{t+1}} \alpha_1 \frac{Y_{t+1}}{K_{t+1} - K_{2,t+1}} &= 0 \\
K_{2,t}; -\frac{1}{C_t} \alpha_1 \frac{Y_t}{K_t - K_{2,t}} + \lambda_t^F \alpha_2 \frac{F_t}{K_{2,t}} &= 0 \\
L_{2,t}; -\frac{1}{C_t} \frac{\left(1 - \alpha_1 - \nu_1\right) Y_t}{L_t - L_{2,t}} + \lambda_t^F \left(1 - \alpha_2 - \nu_2\right) \frac{F_t}{L_{2,t}} &= 0 \\
B_t; \frac{1}{C_t} \nu_1 \frac{Y_t}{O_t + B_t} - \theta \frac{1}{F_t - B_t} + \lambda_t^B &= 0 \\
O_t; \frac{1}{C_t} \nu_1 \frac{Y_t}{O_t + B_t} + \lambda_t^O - \beta^{-t} \lambda^R &= 0 \\
F_t; \frac{\theta}{F_t - B_t} - \lambda_t^F &= 0 \\
\lambda_t^B B_t &= 0 \\
\lambda_t^O O_t &= 0
\end{aligned} \tag{5}$$

The current-value Kuhn-Tucker multipliers λ_t^B and λ_t^O are strictly positive if the constraints $B_t \geq 0$ and $O_t \geq 0$ binds. For the shadow value on the resource constraint of oil it holds that $\lambda^R > 0$ since oil is always valuable for production and has no extraction cost.

Claim 1. $\lambda_t^B \lambda_t^O = 0$.

Proof. Suppose otherwise that both λ_t^B and $\lambda_t^O > 0$, then $B_t = O_t = 0$ implying $Y_t = 0 = C_t$ and therefore utility is minus infinity. This is not part of an optimal plan provided $K_0 > 0$. ■

Proposition 2. Depending of the values of the Kuhn-Tucker multipliers, we can distinguish the following three phases.

if $\lambda_t^B > 0$ and $\lambda_t^O = 0$, the economy is in phase 2(Fossil). Oil is used ($O_t > 0$) but no biocarbon is burnt ($B_t = 0$).

If $\lambda_t^B = \lambda_t^O = 0$, the economy is in phase 3(Mixed). Oil is used ($O_t > 0$) and some biocarbon is burnt ($B_t > 0$).

If $\lambda_t^B = 0$ and $\lambda_t^O > 0$, the economy is in phase 4(biocarbon). Oil is not used ($O_t = 0$) but biocarbon is burnt ($B_t > 0$).

The following subsections characterize the three phases.

Phase 2

Consider first phase 2, when $\lambda_t^B > 0$ and $\lambda_t^O = 0$. Oil is used ($O_t > 0$) but no biocarbon is used for energy ($B_t = 0$).

Using the first-order condition for B_t to substitute for λ_t^B in the first-order condition for $K_{2,t}$ and $L_{2,t}$,while noting that $B_t = 0$, yields

$$\begin{aligned} \frac{K_t - K_{2,t}}{K_t} &= \frac{1}{1 + \theta \frac{\alpha_2}{\alpha_1}(1 - s_t)} \\ \frac{L_t - L_{2,t}}{L_t} &= \frac{1}{1 + \theta \frac{(1-\alpha_2-\nu_2)}{(1-\alpha_1-\nu_1)}(1 - s_t)} \end{aligned} \tag{6}$$

where s_t is the savings rate, defined as $s_t = \frac{Y_t - C_t}{Y_t}$.Thus, as long as the savings rate is constant, a fixed share of capital and labor, respectively, is used in final good production. Now let us characterize the dynamic decisions, i.e., O_t and K_{t+1}.Using the first line of (6) shifted one period ahead in the first-order condition for K_{t+1} yields

$$\frac{s_t}{1-s_t} = \alpha_1 \beta \frac{1+\theta \frac{\alpha_2}{\alpha_1}\left(1-s_{t+1}\right)}{1-s_{t+1}}$$

This is satisfied for a constant savings rate

$$s_F \equiv \beta \frac{\alpha_1 + \theta \alpha_2}{1+\beta\theta\alpha_2} \tag{7}$$

which is the savings rate in period t if the economy is in phase F in period t and $t+1$. By assumption, $B_t = 0$ and $\lambda_t^B > 0$ From now on we use subscripts F for fossil to denote equilibrium variables relevant in phase 2, subscripts M for mixed for equilibrium variables in phase 3 and subscript B for equilibrium variables in phase 4 and 1.

Now turn to the implication of $\lambda_t^B > 0$, namely that it would be valuable to have $B_t < 0$ which we can interpret as a preference for "eating oil". The price of food in such a situation must be higher than the price of energy. To see this, divide the the first-order condition for B_t by the marginal utility of consumption and use the constant savings rate result to arrive at

$$Y_t \left(\frac{\theta\left(1-s_F\right)}{F_t} - \frac{\nu_1}{O_t} \right) = \left(1-s_F\right) Y_t \lambda_t^B > 0. \tag{8}$$

The left-hand side is the difference in the price of food and oil (in terms of the consumption good). To see this, note that $\frac{\theta Y_t (1-s_F)}{F_t} = \frac{\theta C_t}{F_t}$ is the ratio of the marginal utility of food and the consumption good and $\frac{\nu_1 Y_t}{O_t}$ is the marginal productivity.

The FOCs for O_t and O_{t+1} if the economy is in phase F in t and $t+1$, yield

$$\frac{O_{t+1}}{O_t} = \beta. \tag{9}$$

This is a variant of the standard Hotelling result, and implies that oil use is given by $O_t =$

$(1-\beta)\, R_t$.

PROPOSITION 3. In phase 2, the saving rate and the shares of capital and labor allocated to the production of food are all constants given by

$$\begin{aligned} \frac{K_{2,t}}{K_t} &= \kappa_F \equiv \frac{1}{1+\frac{\alpha_1}{\theta\alpha_2(1-s_F)}} \\ \frac{L_{2,t}}{L} &= \Lambda_F \equiv \frac{1}{1+\frac{1-\alpha_1-\nu_1}{\theta(1-\alpha_2-\nu_2)(1-s_F)}} \end{aligned} \tag{10}$$

and (7).

The consequence of the proposition is that there is a balanced growth path in phase 2. Specifically, if the technology trends grow a constant rate, the economy converges to a path with a constant growth rate shared by capital and output given by

$$\gamma_Y = \frac{\gamma_{A_c} + (1-\alpha_1-\nu_1)\,\gamma_L + \nu_1 \ln\beta}{1-\alpha_1}$$

and the growth rate of food production is given by

$$\gamma_F = \gamma_{Af} + \alpha_2\gamma_Y + (1-\alpha_2-\nu_2)\,\gamma_L$$

The price of oil is determined by the ratio $\frac{\nu_1 Y}{O}$, the growth rate of this price is equal to the difference between the growth rates of output and oil use. Defining the growth rate of a variable x,as $\gamma_x \equiv \ln\left(\frac{x_{t+1}}{x_t}\right)$, the price of oil grows at a rate γ_y-$\ln\beta$ (i.e., output growth plus the subjective discount *rate*). Since the savings rate is constant, the growth rate of the food price is equal to $\gamma_y - \gamma_F$. Clearly, this implies that if $-\ln\beta > -\gamma_f$, i.e., if the growth rate of food consumption is higher than minus the subjective discount rate, the price of oil grows faster than the price of food.

It then follows immediately that

PROPOSITION 4. With constant technology growth rates γ_{A_c} and γ_{A_f}, the economy in phase 2 converges to a balanced growth path in which the growth of the price of oil is higher than the growth rate of the price of food if

$$-(1-\alpha_2\nu_1)\ln\beta > -\gamma_{A_f} - \frac{\alpha_2}{1-\alpha_1}\gamma_{A_c} - \left(\frac{\alpha_2}{1-\alpha_1}(1-\alpha_1-\nu_1) + (1-\alpha_2-\nu_2)\right)\gamma_L.$$

Note that the LHS of the inequality in the proposition is positive. Thus a sufficient condition for the growth rate of oil to be higher than that of food is that technological growth and labor endowments have non-negative growth rates. If the condition is satisfied, the sign of the difference of the price and oil and food cannot remain strictly positive forever. Therefore, eventually, λ_t^B cannot remain strictly positive forever and consequently, phase 2, the fossil phase, is transient.

Phase 3

In the phase of mixed use of fossil and biocarbon for energy production, we have an interior solution and neither of the constraints $B_t \geq 0$ and $O_t \geq 0$ is binding, thus $\lambda_t^B = \lambda_t^O = 0$. The first-order condition for B_t then implies that

$$\frac{\nu_1 Y_t}{O_t + B_t} = \theta \frac{C_t}{F_t - B_t} \tag{11}$$

where the LHS is the price fuel, which we denote by P_t^E and the RHS the price of food, denoted P_t^O.

Furthermore, if $\lambda_t^O = \lambda_{t+1}^O = 0$, the condition O_t and O_{t+1} implies that

$$\frac{\nu_1 \frac{Y_{t+1}}{O_{t+1}+B_{t+1}}}{\nu_1 \frac{Y_t}{O_t+B_t}} = \frac{C_{t+1}}{\beta C_t} = \alpha_1 \frac{Y_{t+1}}{K_{t+1} - K_{2,t+1}} \tag{12}$$

where the last equality comes from the Euler equation (the first-order condition for K_{t+1}). The LHS of (12) is the gross growth rate of the price if fuel and the RHS is the gross interest rate. We therefore get the following proposition:

PROPOSITION 5. In phase M, the price of fossil fuel, denoted P_t^O, and biocarbon, denoted P_t^D are equal and grow at the rate of interest, given by the sum of consumption growth and the subjective discount rate, $\gamma_C - \ln\beta$.

In contrast to phase F, the shares of capital and labor allocated to biocarbon production (food and fuel) are not constant and neither is the savings rate. From the first-order conditions for condition for $K_{2,t}$ and $L_{2.t}$ we find that

$$\begin{aligned} \frac{K_{2,t}}{K_t} &= \frac{1}{1+\frac{F_t-B_t}{F_t}\frac{\alpha_1}{\theta\alpha_2(1-s_t)}} \\ \frac{L_{2,t}}{L_t} &= \frac{1}{1+\frac{F_t-B_t}{F_t}\frac{1-\alpha_1-\nu_1}{\theta(1-\alpha_2-\nu_2)(1-s_t)}} \end{aligned} \tag{13}$$

Over time $\frac{F_t-B_t}{F_t}$, i.e., the share of biocarbon production that is used for food falls. This leads to increasing food pirces and an increase in the shares of capital and labor allocated to biocarbon production. Furthermore, the first-order condition for K_{t+1} implies that savings satisfies

$$s_t = \frac{\beta\alpha_1}{(1-s_{t+1})\left(1-\frac{K_{2,t}}{K_t}\right)+\beta\alpha_1}.$$

As we see, a higher $\kappa_{2,t+1}$ yields higher savings in the current period, given next period's savings.

Phase 4

In phase 4, biocarbon is the only source of fuel. We then have from the first-order condition for O_t that

$$\frac{1}{C_t}\nu_1\frac{Y_t}{B_t} = \beta^{-t}\lambda^R - \lambda_t^O \tag{14}$$

Since $\lambda_t^O > 0$, the value of fuel is lower than the current shadow value of the resource constraint ($\beta^{-t}\lambda^R$). The price of fuel in terms of consumption goods $\left(\frac{\nu_1 Y_t}{B_t}\right)$ is therefore lower than the implicit price that would be required for the planner or the market to save oil in the ground for use in period t.

Proceeding as above, we find that a constant savings rate and a constant share of biocarbon used for energy purposes satisfies the relevant first-order conditions. The solutions are

$$\begin{aligned} s_t &= s_B \equiv s_F + \beta \frac{\alpha_2 \nu_1}{1 + \beta\theta\alpha_2} \\ \frac{B_t}{F_t} &= \Phi_B = \frac{\nu_1}{\nu_1 + \theta\left(1 - s_B\right)} \end{aligned}$$

Using this to replace s_t and $\frac{F_t - B_t}{F_t}$ in the first order conditions for $K_{2,t}$ and $L_{2,t}$ yields

$$\begin{aligned} \frac{K_{2,t}}{K_t} &= \kappa_B \equiv \frac{1}{1 + \left(1 - \Phi_B\right)\frac{\alpha_1}{\theta\alpha_2(1-s_B)}} \\ \frac{L_{2,t}}{L_t} &= \Lambda_B \equiv \frac{1}{1 + \left(1 - \Phi_B\right)\frac{1-\alpha_1-\nu_1}{\theta(1-\alpha_2-\nu_2)(1-s_B)}} \end{aligned} \tag{15}$$

Since $\Phi_B > 0$, if follows that $\kappa_B > \kappa_F$ and $\Lambda_B > \Lambda_F$, that is, larger shares of capital and labor are allocated to the sector producing food and energy in phase B than in phase F.

Now use the finding of a constant savings rate in (14), yielding

$$\frac{\nu_1}{\left(1 - s_B\right)} \frac{1}{B_t} = \beta^{-t}\lambda^R - \lambda_t^O \tag{16}$$

From this follows immediately:

Proposition 6. If the economy is in phase 4 in period t, $\frac{\nu_1}{(1-s_B)}\frac{\beta^t}{B_t} < \lambda^R$. Therefore, if $\frac{B_{t+1}}{B_t} \geq \beta$, the economy is in phase 4 in period t+1.

Corollary 7. If $\gamma_{A_f} + \alpha_2\gamma_K + \left(1 - \alpha_2 - \nu_2\right)\gamma_L > \ln\beta$, the condition in the proposition is satisfied.

The economy in phase 4 converges to a constant growth rate with $\gamma_Y = \gamma_K$ if the technological growth rates are constant. Then, the condition in the corollary can be expressed in exogenous parameters. In any case, we see that a positive growth rates of technology, capital and labor are sufficient for the economy to indefinitely remain in phase 4.

Let us finally discuss the existence and timing of the three phases. We have shown that 1) under mild conditions, phase 2 is transient, 2) at least one of phase 2 and 3 must exist, 3) under some mild conditions, phase 4 or B is absorbing. Also phase 3 is transient.[7] Then, the economy starts in phase 2 or 3 and necessarily ends up in phase 4. Whether phase 2 exists depends on the size of initial oil reserves. With sufficiently large oil reserves, the initial oil price is below the price of food, i.e., there is an initial phase 2.[8] Finally, we note that we can easily add a phase 1, in which oil (or the technology to use oil) has not yet been discovered. This is again phase B. In all respects, it is identical to phase 4. In that sense, the fossil era, consisting of phase 2 and 3 (or only a phase 3), is a parenthesis, just an episode in history.

Abstracting from growth in technology and labor, the price of food develops scematically as in shown in figure 1. When the fossil technology is discovered, land, capital and labor used for the production of fuel can be realeased for other purposes. This reduces the scarcity of food, and the food price falls. This, in turn, leads to a reallocation of labor and capital to the production of consumption goods. At the end of phase2, the price of fuel has reached the price of food and both then during phase 3 rise at the rate of interest. Eventually, all oil is used and the economy is in phase 4.

Quantitative analysis

A key policy issue is how much the price of food will rise during the phase of rapid growth that occurs in phase 3. Let us now perform a simple exercise to calculate this. Let us

[7] Given the parametric assumptions, it is necessary that an increasing share of biocarbon is used for fossil fuel in order to generate food prices that increase at the rate of interest rate. Eventually, the rate of food price increase falls below the interest rate and the reallocation stops. At this time, all oil is used up and phase B is entered. **Proof remains.**

[8] Idea of proof: Suppose the economy starts with a capital stock K_0 and enters phase M immediately. Our results then imply that we can calculate the amount of oil that will be used in phase M. If the initial oil reserve R_0 is larger than this amonut, a phase F must exist. **Proof remains.**

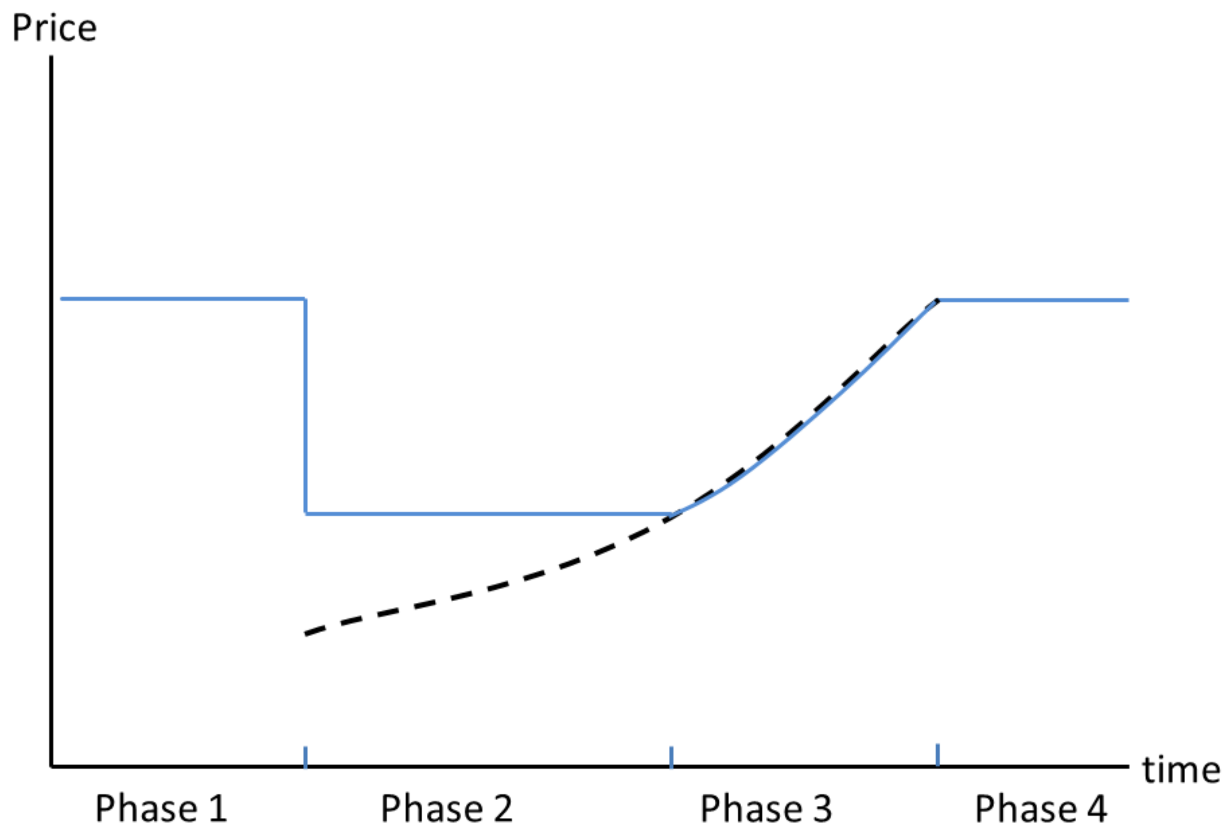

Figure 1: Qualitative description of how the food price (solid curve) and oil price (dashed curve) developes over time during the four phases.

first normalize the aggregate labor supply to unity and disregard population growth[9]. Also normalize the supply of land to unity. During phase 2, the food price is then given by

$$P_t^D = \theta \frac{(1 - s_F) Y_t}{F_t} \tag{17}$$

To calculate this price, we need to know energy use at the end of phase 2. Using the fact that what defines the shift to phase 3 is that the energy price has risen to the level of the food price we set the price of energy equal to the price of food and solve for oil use giving

$$O_{t_F} = \frac{\nu_1 F_{t_F}}{\theta (1 - s_F)}$$

where we let t_F denote the last period in phase F.[10]

Using this, the production functions and the optimal allocation rules and setting $\alpha_1 = \alpha_2 (1 - \nu_1) = \alpha$ yields a food price at the end of phase 2 given by[11]

$$\frac{A_{c,t_F}}{A_{f,t_F}^{1-\nu_1}} \theta (1 - s_F) \left(\frac{1 - \kappa_F}{\kappa_F} \right)^{\alpha} \frac{(1 - \Lambda_F)^{1-\alpha_1-\nu_1}}{\left(\Lambda_F^{1-\alpha_2-\nu_2}\right)^{1-\nu_1}} \left(\frac{\nu_1}{\theta (1 - s_F)} \right)^{\nu_1}. \tag{18}$$

Let t' denoted a time period in phase 4. The food and energy prices, $P_{t'}^D$ and $P_{t'}^E$ are equal in phase 4, since they both come from biocarbon and the prices are given by the marginal product of biocarbon in consumption good production;

$$P_{t'}^D = P_{t'}^E = \nu_1 \frac{Y_{t'}}{B_t'}$$

[9]Of course, poulation growth may itself be important for the development of food prices. But this is not the focus of this paper.

[10]It should be noted that this involves a slight approximation due to the discrete time assumption. In the last period of phase 2, the price is weakly below the food price and in the first period of phase 3, biocarbon use for energy purposes is small. The approximation is built on the assumption that either the use of biocarbon for enery in the first period of phase 3 is small or that the food price is close to the energy price in the last period of phase 2.

[11]The parametric assumption $\alpha_1 = \alpha_2 (1 - \nu_1)$ implies that the capital stock has no effect on the price of food. Note that it implies that the income share to capital net of energy costs is the same in the two sectors.

which under the assumptions above become

$$\frac{A_{c,t'}}{A_{f,t'}^{1-\nu_1}}\nu_1\left(\frac{1-\kappa_B}{\kappa_B}\right)^{\alpha}\frac{((1-\Lambda_B))^{1-\alpha_1-\nu_1}(\Phi_B)^{\nu_1}}{\Phi_B\left(\Lambda_B^{1-\alpha_2-\nu_2}\right)^{1-\nu_1}} \tag{19}$$

As we see, the food price at the end of phase 2 (given by (18)) and the food price in phase 4 (given by (19)) both depend on the relation between the two technology trends. We also see that, if $\gamma_{A_c} > (<)\,\gamma_{A_f}(1-\nu_1)$, both relative prices increase (decrease). The intuition for why a higher rate of technical change in the biocarbon sector is required for constant relative prices is that biocarbon is used as an input in the production of consumption goods. Finally, in order to separate the endogenous price effect from that coming from exogenous technology trends, let us set $\gamma_{A_c} = \gamma_{A_f}(1-\nu_1)$. It then follows that $\frac{A_{c,t_F}}{A_{f,t_F}^{1-\nu_1}}/\frac{A_{c,t'}}{A_{f,t'}^{1-\nu_1}} = 1$ and that the effect on the price of food due to the transition from phase 2 to phase 4 is given by the ratio

$$\frac{P_{t'}^D}{P_{t_F}^D} = \left(\frac{\nu_1}{\theta(1-s_F)\Phi_B}\right)^{1-\nu_1}\left(\frac{1-\kappa_B}{1-\kappa_F}\frac{\kappa_F}{\kappa_B}\right)^{\alpha}\left(\frac{1-\Lambda_B}{1-\Lambda_F}\right)^{1-\alpha-\nu_1}\left(\frac{\Lambda_F}{\Lambda_B}\right)^{\left(1-\frac{\alpha}{1-\nu_1}-\nu_2\right)(1-\nu_1)}$$

which using the definitions above is completely expressed in terms of exogenous parameters.

Let us now put parameter values to this expression. We set $\alpha = 0.3, \beta = 0.98^{10}$ and study how the other parameters affect the price ratio. As a benchmark we set the income share of energy, ν_1 to 0.05, $\theta = 0.1$, implying an income share of the biocarbon sector of $\frac{1}{11}$ and the income share of land, ν_2, to 0.2.

At the benchmark values, the price increase $\frac{P_{t'}^D}{P_{t_F}^D} - 1$, is quite modest, at 9.4%. Recalling that the price of food (and fuel) increase by the rate of interest, this would imply a short transitional phase 3 for a reasonably calibrated interest rate. Over this period, capital and labor has to be reallocated to the biocarbon sector. The share of capital in the biocarbon sector increases from $\kappa_F = 7.2\%$ to $\kappa_B = 11.4\%$. For labor, the corresponding figures are $\Lambda_F = 5.2\%$ and $\Lambda_B = 8.3\%$. In particular for labor, this may represent reallocation with significant costs. The share of biocarbon used for energy in final goods production increase from zero to 40.9%. The savings rate increases slightly, from 26.4% to 27.7%.

It should be noted that we have assumed a constant aggregate labor supply. If the income share of labor net of energy costs in the consumption sector $(1 - \alpha_1 - \nu_1) / (1 - \nu_1)$ is larger than the income share of labor in the biocarbon sector $(1 - \alpha_2 - \nu_2)$, increasing labor supply has a positive impact on the food price. The reason is that if the labor share is large in producing consumption good, an increased labor supply makes the consumption good less scarce and thus food and fuel more scarce. The growth rate of the price due to this effect is given by the difference in income shares times the growth rate of labor supply, which arguably is a small number, unless the transition is long.

In the following graphs, we analyze how the effect of the transition on food prices responds to variation in the parameters. Specifically, we plot the price increase against ν_1, θ and ν_2,respectively, holding the other parameters at their benchmark values.

Figure 2 shows that the price increase is increasing in the income share of energy in consumption goods production. The sensitivity is not very large – a doubling of the income share to 10% leads to a price increase of 15.0%.

Figure 3 shows that the price increase falls in the income share of food. With a high income share of food, a large share of capital and labor is allocated to the biocarbon sector already in phase 2 and the extra increase in the demand for biocarbon therefore have small effects. At low income shares, the price increase can be quite substantial, implying a long transition period.[12] We should note that although the share of income spent on food is higher than the benchmark 1/11, a substantial share of that is spent on transportation and services, which is not really part of the biocarbon production. An arghably more reasonable value to calibrate θ to is the share of the agriculture sector in GDP. We therefore consider a low value of θ to be realistic.

Figure 4 shows the effect of varying the income share of land in agriculture. The price increase is higher the higher the income share of land. This is due to the fact that a higher

[12] Recall that during the transitional period M, food prices rise at the rate of interest. Given the rate of interest, a higher price increase takes longer time.

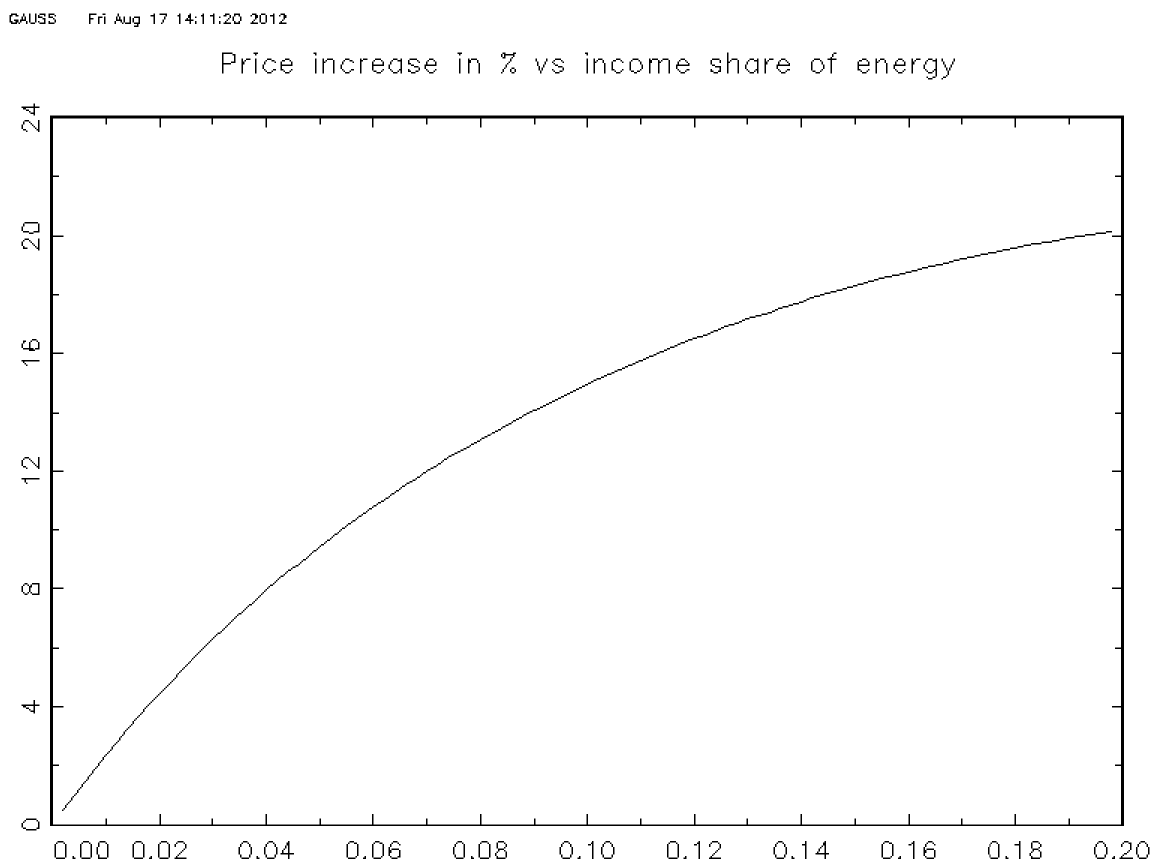

Figure 2: Increase in relative price of food due to transition to phase 4. Income share of energy in consumption goods sector (ν_1) on x-axis.

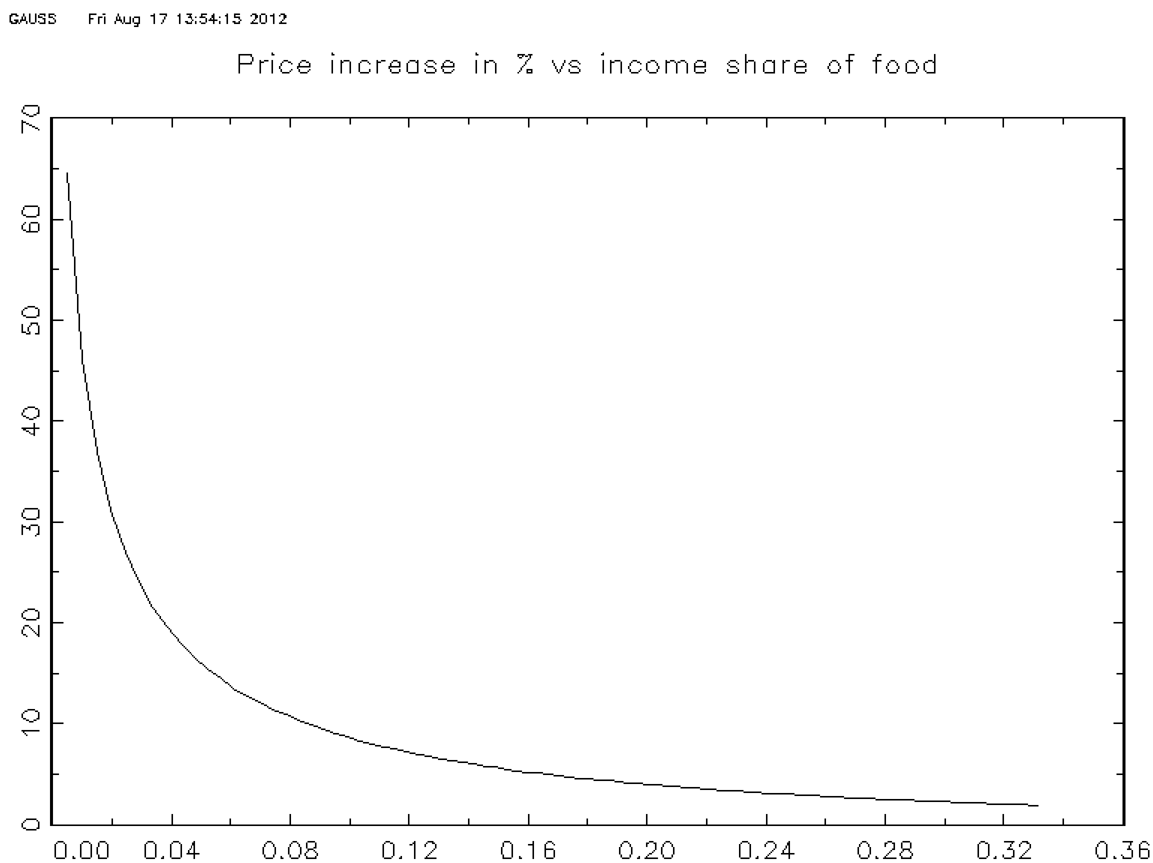

Figure 3: Increase in relative price of food due to transition to phase 4. Consumption share of agricultural sector ($\theta/(1+\theta)$) on x-axis.

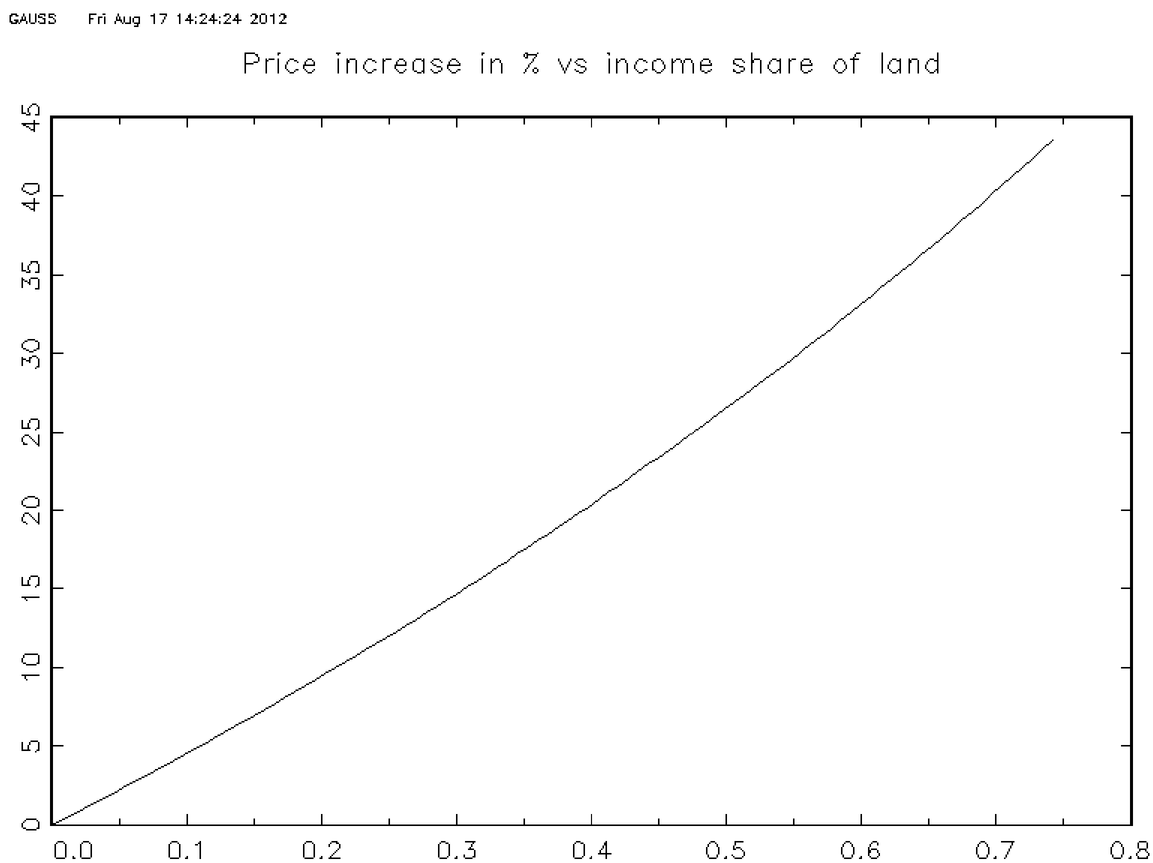

Figure 4: Increase in relative price of food due to transition to phase 4. Income share of land (ν_2) on x-axis.

income share of the fixed factor implies faster declining marginal products when output is increased by moving labor and capital to the sector. Although standard estimates of this income share are fairly low, we may consider higher values as reasonable if we think that additional reasons for a falling marginal product may exist.[13] Here we are thinking of the fact that a higher demand for biocarbon leads to less productive land being taken into use.

Finally, let us conclude by noting that the model is able to generate quite sizeable increases in the food price if the income share of food is low and the energy and land share are high. Setting $\theta = 0.05$, $\nu_1 = 0.08$, and $\nu_2 = 0.5$, which arguably is somewhat extreme, gives a price increase of 67.1%.

Concluding remarks

In a nutshell our theory of industrialization is simple. Since its very existence mankind has lived on biocarbon. Then, all of a sudden, it found a way to dig holes in the ground and use the fossil carbon stored there some 300 million years previously. This discovery triggered rapid industrial growth and improved general levels of nutrition, as it meant that all of the arable land could be used for the production of food. When fossil carbon became scarcer mankind decided to reserve some of it for later use and return a share of the land to energy production. Finally, when fossil carbon is used up, mankind will have to go back to living on biocarbon only.

In our opinion, the model demonstrates the power of neoclassical economics since it is able to describe and explain fundamental phases in the history of industrialization while satisfying first principles and the main theorem of welfare economics.

In terms of prices, a qualitative description of the four phases, where we abstract from effects due to technology trends is as follows. In the pre-industrial phase (1), the food price is constant . After inventing a way to tap fossil carbon, phase (2) begins with a sharp drop in

[13] Land share in agriculture is 0.18 in "Measuring Factor Income Shares at the Sector Level", Akos Valentinyi,Berthold Herrendorf, (2007)

the food price as land previously devoted to grow fodder is now freed up for the production of food. Since the energy price in this phase is even lower than the food price, and fossil carbon cannot be eaten, the economy is driven into a corner solution with no land being used for energy production. The food price now stays constant, but the energy price rises according to the Hotelling rule. When it reaches the food price, phase (3) starts and the energy price pulls the food along. In this phase a steadily increasing fraction of agricultural output is devoted to energy production and a steadily increasing fraction of the available capital and labor is shifted from manufacturing to agriculture. The price of food now also grows in line with the Hotelling rule, i.e. at the rate of interest. If the price of fossil fuel changes, for example due to the discovery of new oil or demand chocks, the price of food would also change. The phase ends in finite time with an exhaustion of the stock of fossil fuel, and the economy enters phase (4) where all the energy comes from bio sources. The price of biocarbon in this phase is a constant and the economy follows a steady state growth path.

Given the interest rate, phase (3) is longer the higher is the price of food in phase (4) relative to phase (2). (In section , we derive closed form expressions for this relative price.) We show, in particular, that a higher income share of energy in the production of consumption goods, a higher income share of land in the biocarbon production and a lower share of income spent by consumers on food leads to a higher increase in the price of food during the transitional phase (3).

One of our findings is that biocarbon in our model acts as a semi-flexible backstop technology resulting in exhaustion of the resource in a finite time. It is not a hard backstop technology that offers energy at a fixed price. After all, during phase (3) the price of biocarbon is permanently rising according to the Hotelling rule, even though it is not an exhaustible resource. However, since a phase of balanced growth without the use of fossil carbon in which the biocarbon price remains constant over time is always available, the existence of such a phase in itself takes the role of a backstop yielding the usual results, in particular the full exhaustion of the resource in finite time.

With the Tortilla crisis, the world economy has just now entered phase (3). After a long period of falling short of the food price, the fossil carbon price has finally reached the same level as it and is now pulling the food price along, enforcing massive reallocations of labor and capital from industry to agriculture and of food from the table to the tank. While all of this is Pareto optimal, we warn the reader that this does not, of course, mean that distributional goals are met.

Due to our assumption of a representative agent, our model does not lend itself well to the distributional question raised by the Tortilla Crisis. Due to the homotheticity of the preferences assumed, the model is compatible with an uneven allocation of property rights in land, capital, labor and the stock of the fossil fuel resource, provided everyone holds these endowments in equal proportions.

However, this naturally is not true in reality. Thus, if the price of food in phase (3), which mankind has probably just entered, keeps rising, this will have problematic implications for the well-being of the poor, as they have another endowment mix with less land, less initial capital and a lower stock of fossil carbon and relatively more labor than other agents. The entry into the mixed post-industrial phase will therefore have problematic distributional implications. We can only touch on this problem and encourage researchers to investigate this issue in the future.

One of the questions to be asked in a distributional variant of our model would be which policy tools might serve the distributional goals, and what welfare losses in the Pareto sense such tools would involve, if any. There is a broad consensus that redistribution via cash flow taxes or, equivalently, a once-and-for all reallocation of endowments would, if unforeseen, be possible without incurring welfare losses. It would be useful in our opinion to study whether land reforms giving the poor entitlements to land might not be a practical way to solve the redistribution problem, without sacrificing overall economic efficiency and welfare.

For a deeper policy analysis with taxes and similar instruments, it would be useful, necessary and possible to reformulate our model as an explicit market model, whereby households plan their consumption paths with financial wealth constraints and companies maximize their market value assuming that they will not be able to affect the time paths of market prices through their own actions.

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