The Contributions of Mathematics in Modeling Economic Growth

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Sept. 3, 2012
Nobel Foundation Symposium on Growth and Development

The Transition to Modern Growth

- Aggression during Pleistocene selected humans with moral systems that were “groupish” as well as “selfish”
- 11,600 BP, warm, stable, climate of Holocene makes sedentary agriculture possible
- Neolithic revolution follows in multiple independent sites
- Groupishness and flexible norms let scale of cooperation expand to larger urban groups

The Transition to Modern Growth

- During Pleistocene, humans also developed capacity for using reasoning to persuade in games with each other
- With neolithic revolution, capacity for persuasion combined with flexible norms leads increasingly to use of reasoning in a game against nature; goal is truth not persuasion
- In the cultural evolution of norms (NOT biological evolution of genes), the gains from sharing nonrival discoveries that are true and useful create strong selection pressure for larger group size

The Transition to Modern Growth

- Human tendency to absorb norms about right and wrong from actions of others creates potential for rapid cultural evolution
- Allowed modern systems of urban life and scientific inquiry even in creatures with a brain tuned to life in the Pleistocene
- However, it may take collective action to maintain the norms that define the in-group in an inclusive way and that promote commitment to truth over persuasion
- True in economic life and economic inquiry

Norms of Science

- With Pleistocene brains, constant risk of
  - segregation into small groups
  - persuasion with no anchor in truth
- Resist with
  - norms of inclusion and engagement
  - logic of math, confrontation with evidence

Norms of Science

- Fermi: “Science is a process for resolving disagreement”
- In economics, rate of convergence seems slow
- But use of mathematics is surely not the source of the problem
- Groupishness, and resulting failure to engage, might be

Romer 1986

$$Y = f(K) \text{ with } f'' > 0$$

$$\dot{K} = Kg(sY) \text{ with } g'' < 0 \text{ and bounded}$$

Y - Output $\quad$ K - Knowledge

Implications:

- Growth rates can increase
- Initial differences can grow
- Shocks can be amplified by endogenous responses

Romer 1986

In retrospect, lots of attention to mathematical foundations:

- Characterize solution:

$$D_1 F(z, z) = 0$$

- Easier to prove existence in continuous time
- Problems with infinite horizon optimization, Econometrica, 1986
- "Lost in Hilbert space?"

Romer 1986

Problems with model:

- Wrong in the sense that it does not fit the long time series evidence (an noted by Chad Jones, Acemoglu et al, Mokyr, ...)

- Incomplete in sense that it does not model interactions between different economies

- Relies on a magic assumption, not just simplifying assumptions

Romer 1986

$$Y = F(k, x; K)$$

k - excludable knowledge

K - nonexcludable knowledge

x - rival inputs

Magic Assumption:

$$Y = F(\cdot, \cdot; K)$$ homogenous of degree 1

Violates basic physics of replication

Romer 1986

$$Y = F(k, x; K)$$

k - excludable knowledge

K - nonexcludable knowledge

x - rival inputs

- Attention to the math exposes magic assumptions to scrutiny

- E.g. Stiglitz-Dasgupta (1988) on inconsistency between external increasing returns and replication argument

Romer 1987

$$Y = L\int_0^A g\left(\frac{x(i)}{L}\right)\, di$$

$$= L^{1-\alpha}\int_0^A x(i)^{\alpha}\, di$$

- Production is homogeneous of degree 1 in rival inputs
- 100% excludability of all rival and nonrival goods

Romer 1987

$$Y = L\int_0^A g\left(\frac{x(i)}{L}\right)\, di$$

$$= L^{1-\alpha}\int_0^A x(i)^{\alpha}\, di$$

- Note transition to specific functional forms

- Has constant rate of growth that is increasing in L

Romer 1990

$$Y = H_Y^{\alpha} L^{\beta} \int_0^A x(i)^{1-\alpha-\beta}\, di$$

$$\dot{A} = \delta H_A^1 A^1$$

- A is nonexcludable in production of new A
- Growth rate is increasing in stock of H
- Coefficients = *1* in equation for growth of A are simplifying assumptions
- But inconsistent with data (as per Jones)

Romer 1990

$$Y = H_Y^{\alpha} L^{\beta} \int_0^A x(i)^{1-\alpha-\beta}\, di$$

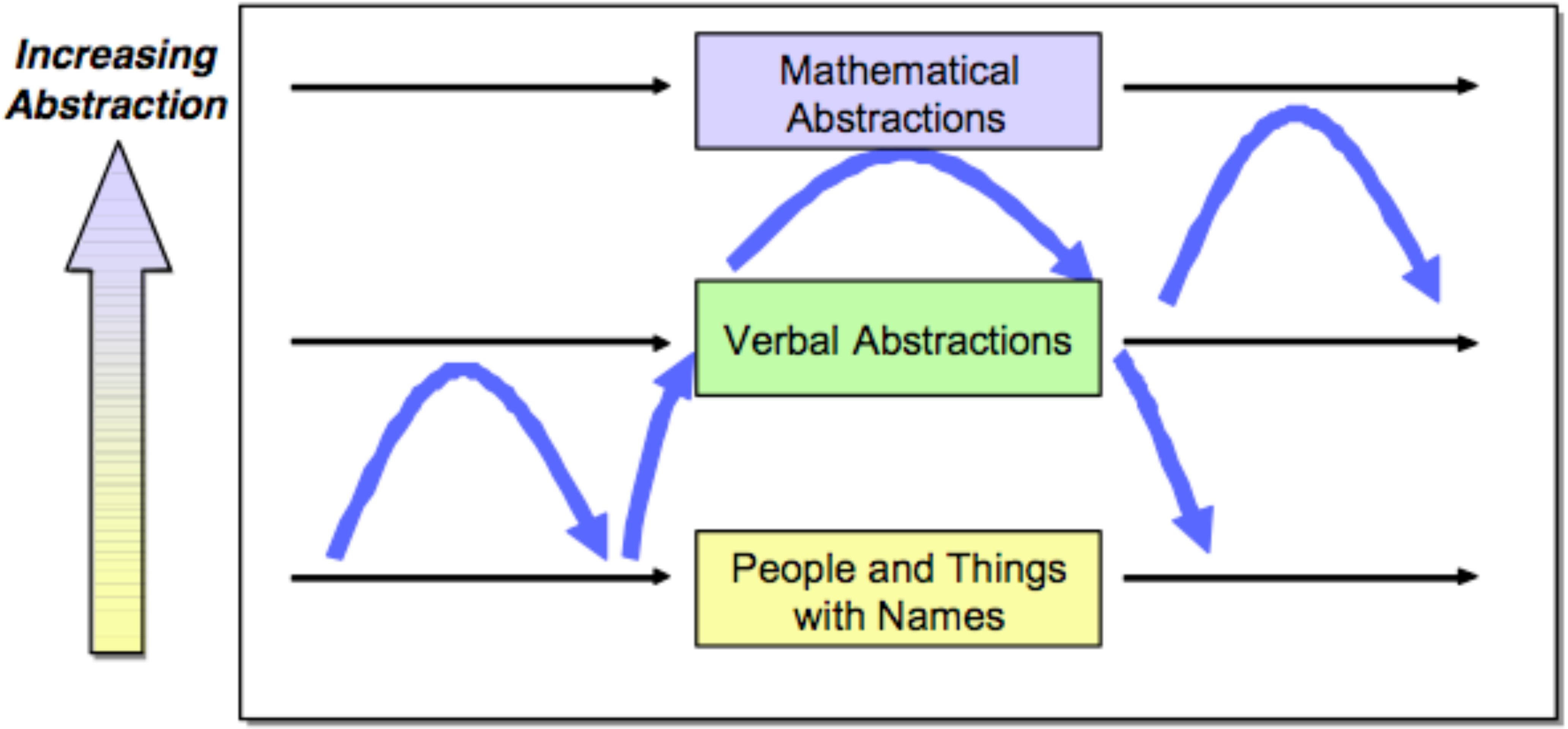
$$\dot{A} = \delta H_A^1 A^1$$

- Excludability allows flows of A between countries inside the model and analysis of trade and growth

- Exponents in equation for growth of A determine growth versus level effects

- Large gains from globalization either way; scale linearly with population

Are Nonconvexities Important for Understanding Growth (1990b)



Unification made possible by nonrival goods

- Growth, Development, Tech Policy, Public Finance
- Increasing rates of growth
- Globalization and gains from trade that are not exhausted at any finite population size
- Urbanization
- Importance of communication (speech, writing, printing, digital transmission and storage)
- Cost advantages of exchangeable parts or “copy exactly” in manufacturing
- Why humans can be non-rivals

Math ⟷ Words

Words made precise by math:

- Public good
- Human capital
- Rivalry and excludability
- Creative destruction

Words where math uncovers confusion:

- Price taking competition with external increasing returns
- Nonpecuniary externalities
- Marshalian Rents that induce innovation

Confronting the Evidence

Alternative Models of Speeding Up

$Y = AX^{\beta}L^{1-\beta}$ X - land

$\dot{A} = \alpha L$ - A nonrival, nonexcludable

$Y / L = \overline{y}$ - Malthusian subsistence

$\dot{A} = \alpha A^{1/\beta}$

- Ron Lee (1988), Kremer (1993)
- Increasing rate of growth associated with increasing size of population

Alternative Models of Speeding Up

- Add demographic transition, per capita economic growth depends on population growth

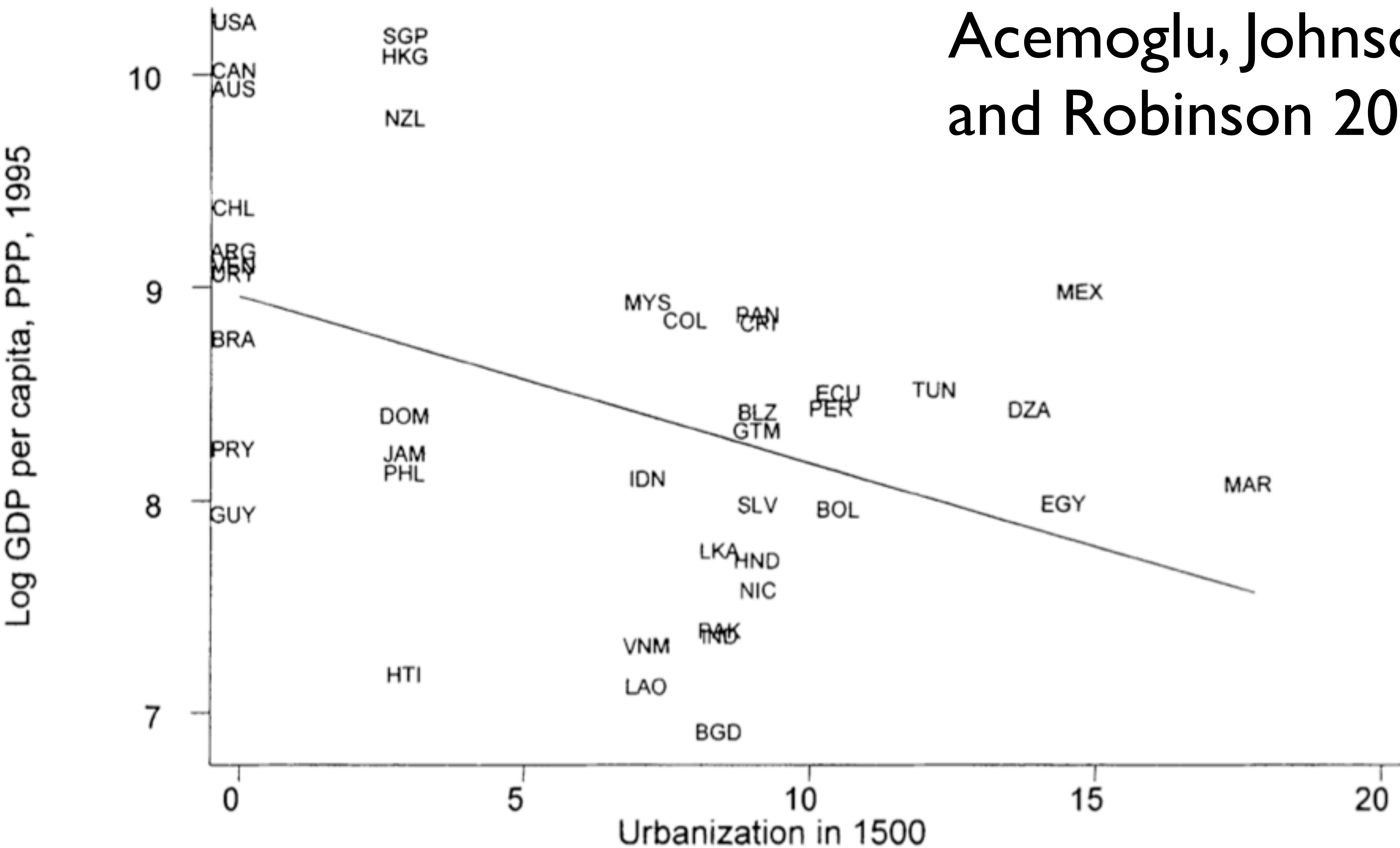
- Rate of growth increasing in rate of growth of the population

- Arrow, Learning by Doing: nonexcludable nonrival

- Jones, separate role for human capital, partial excludability of nonrival goods, connection to literature on social vs. private rates of return

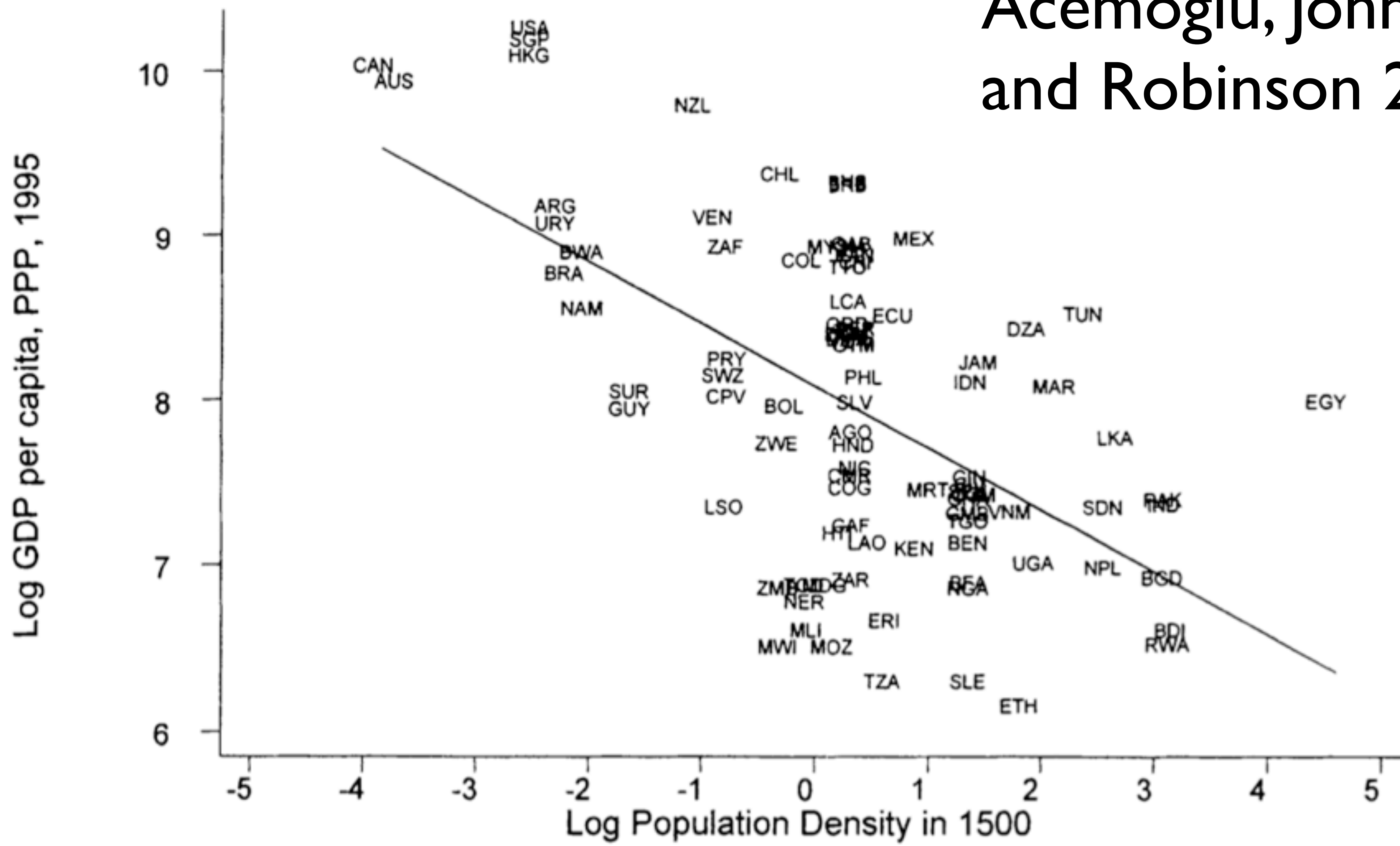
Speeding up vs Cardwell's Law

Acemoglu, Johnson, and Robinson 2002

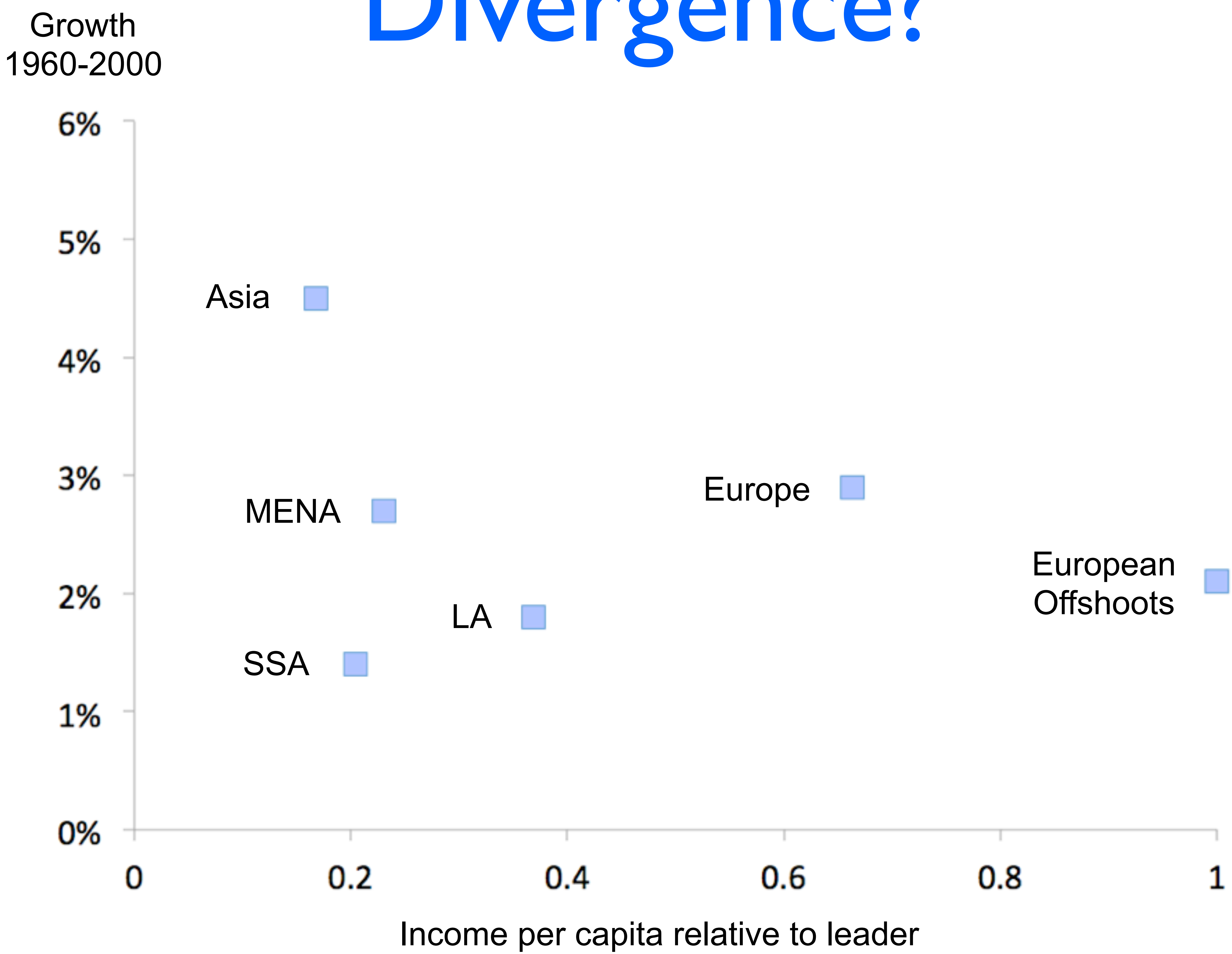


Speeding up vs Cardwell's Law

Acemoglu, Johnson, and Robinson 2002

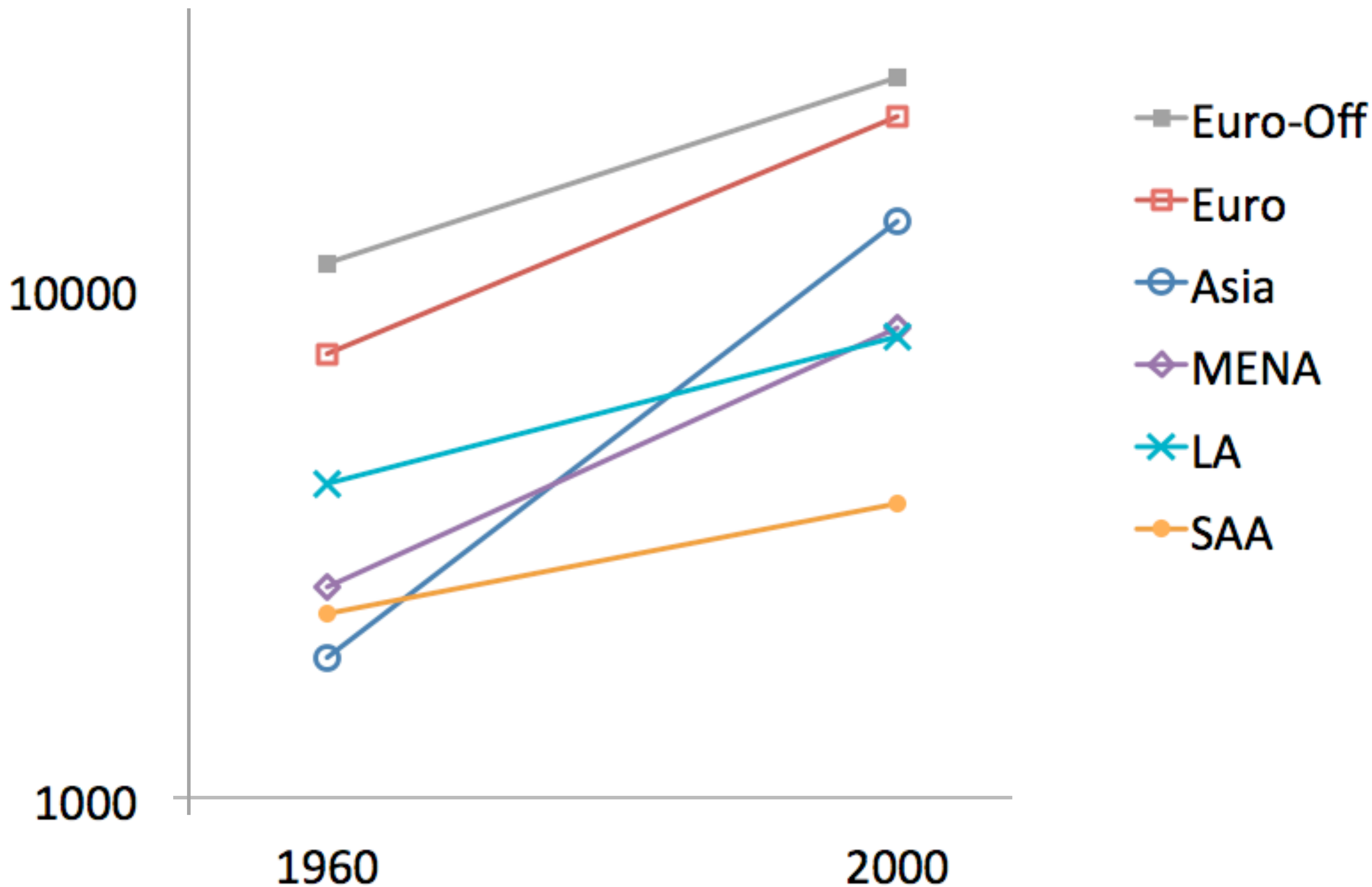


Divergence?

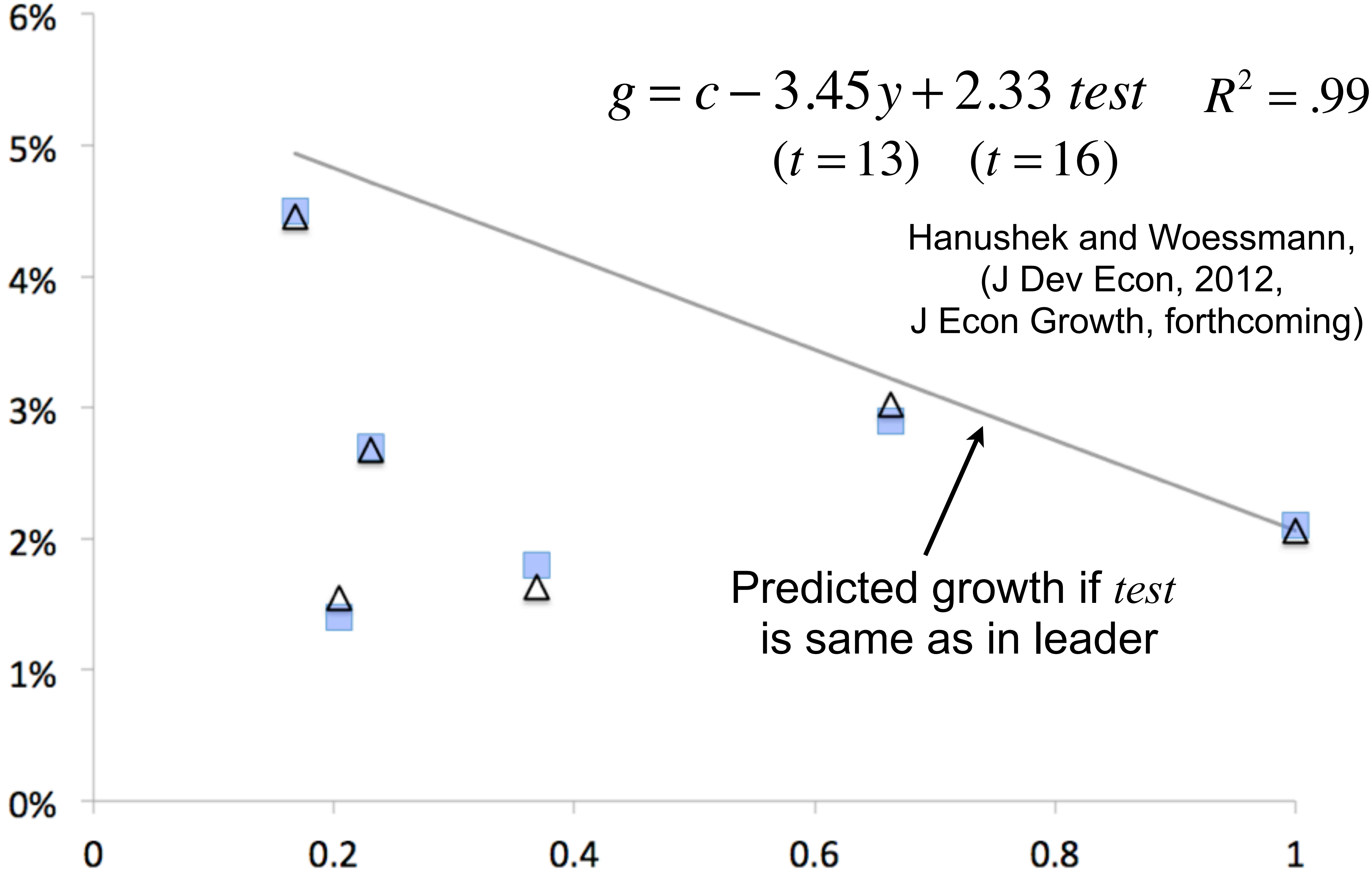


Growth
1960-2000
6%
5%
4%
3%
2%
1%
0%
Asia
MENA
SSA
LA
Europe
European
Offshoots
0
0.2
0.4
0.6
0.8
1
Income per capita relative to leader

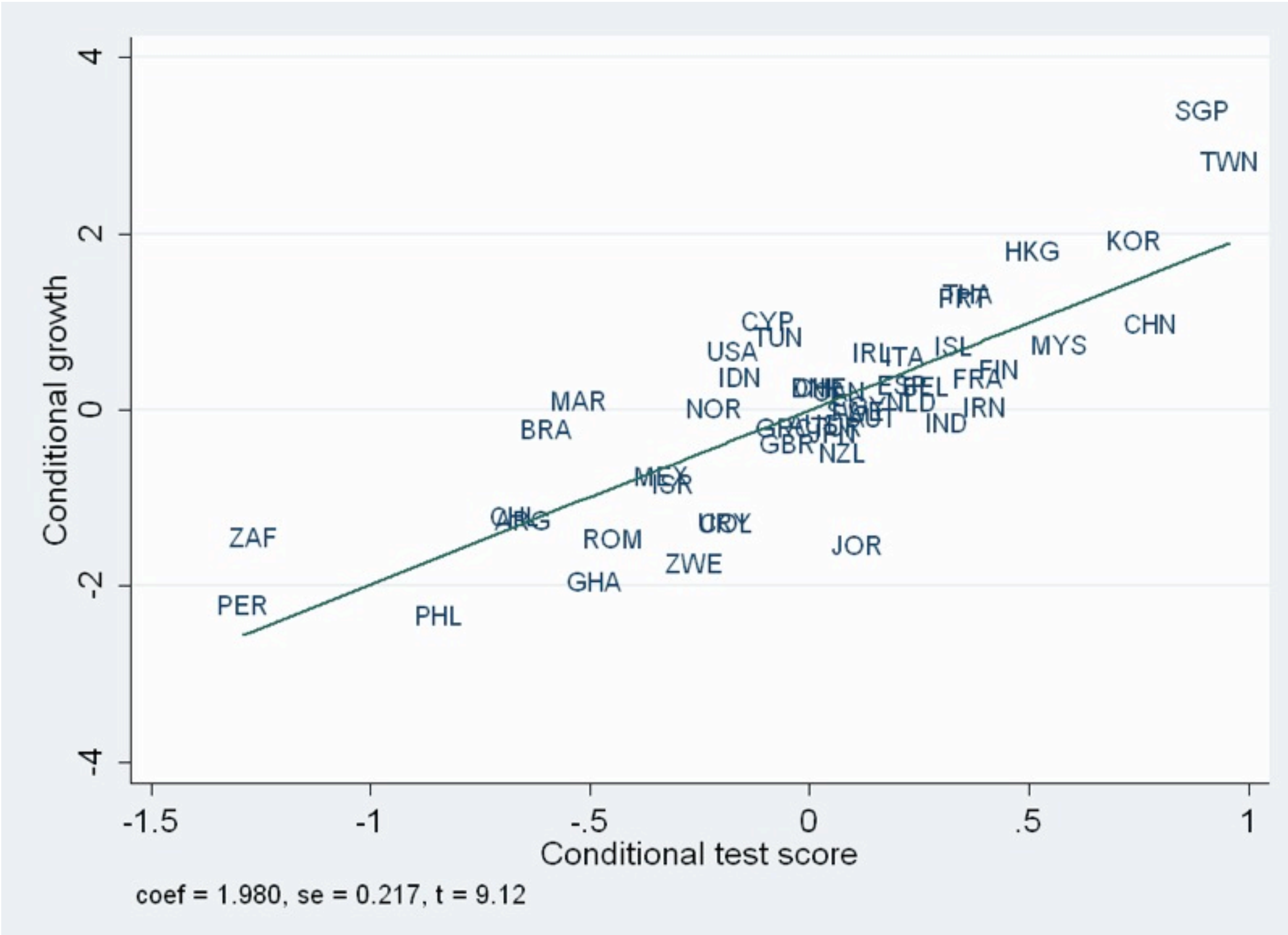
No σ convergence



Strong β Convergence



Hanushek and Woessmann,
(J Econ Growth, forthcoming)



Looking Ahead

Need a workable model:

- with at least 2 driving forces: technology and X
- X must unpack (and clarify) the concept of institutions and endogenize their dynamics
- dynamics must allow for extreme persistence of wildly inefficient equilibria
- but also for occasional rapid changes in X and improvements in efficiency

Looking Ahead

Conjectures:

- X = norms

- norms enter preferences following Becker and Murphy

- mathematics will clarify such words as norms, rules, and institutions

- mathematics will thereby facilitate resolution of scientific disagreement

Looking Ahead

Prediction: Groupish claims about methodology will prevent economists who disagree from fully engaging in the conversation needed to resolve disagreement.

"That's not the way we do things

- We don't use monopolistic competition
- We don't theorize about preferences"